How to validate a method in the mid-frequency range?

Up2HF Summer School

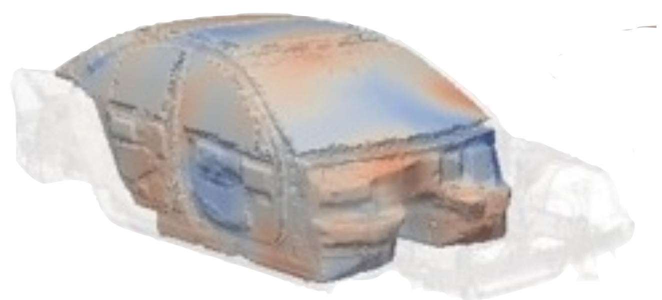
UNIVERSITY OF CAMBRIDGE

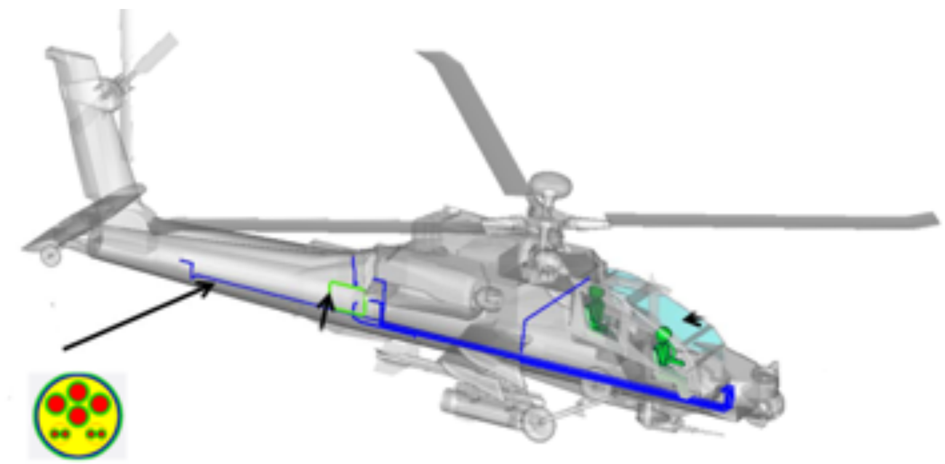
Louis Kovalevsky

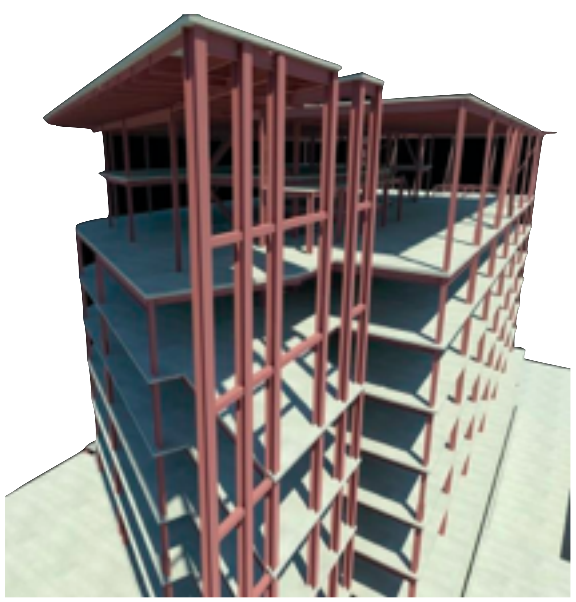
1) brief overview of numerical methods for VA

2) how to validate a numerical method?

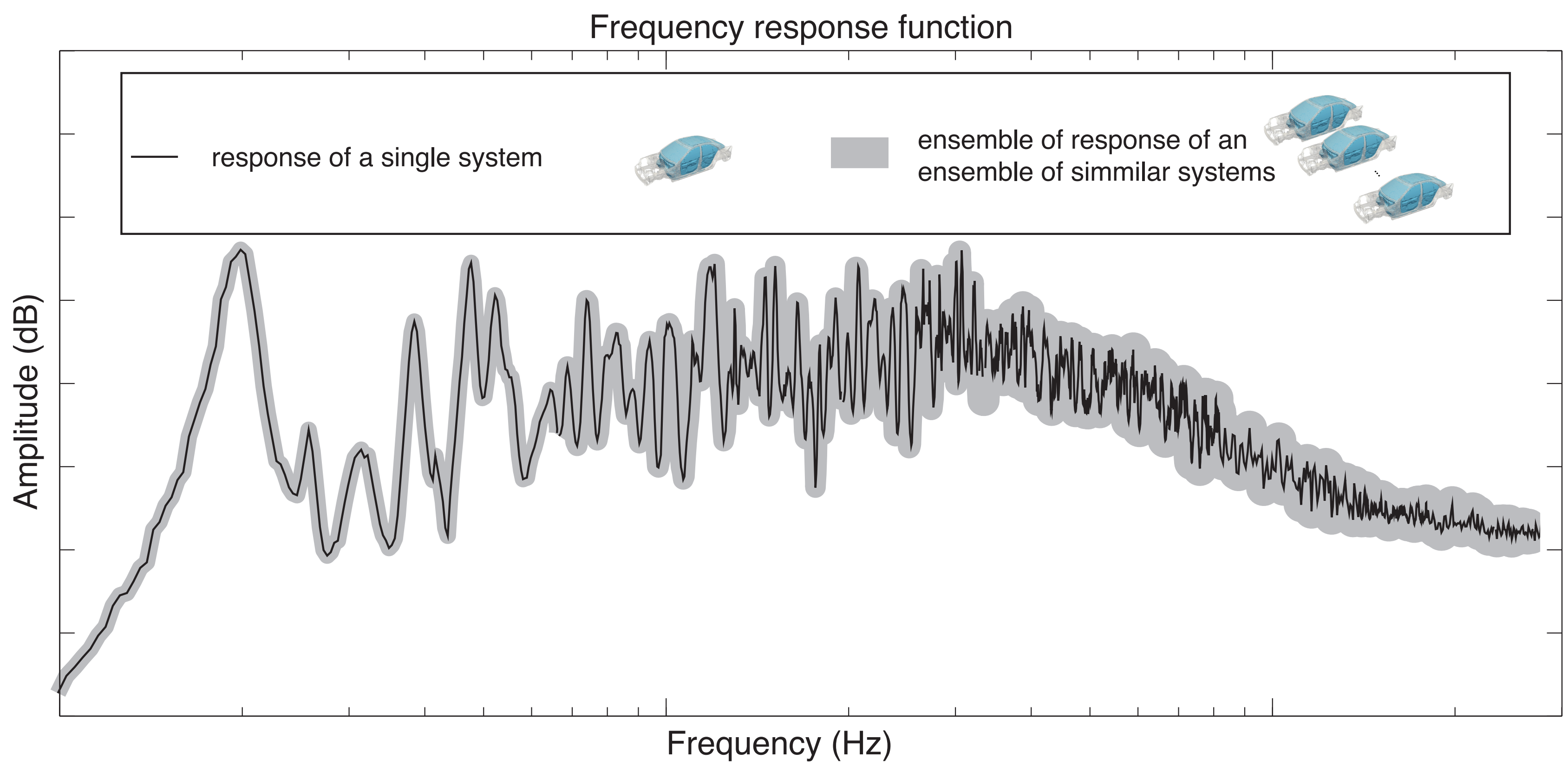
Vibration problems



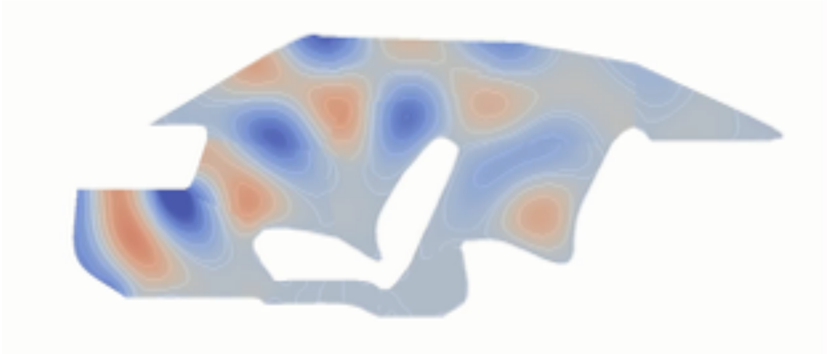




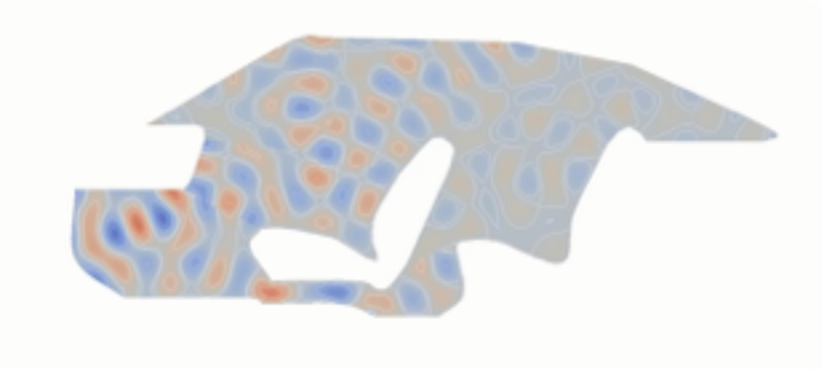
Vibration problems



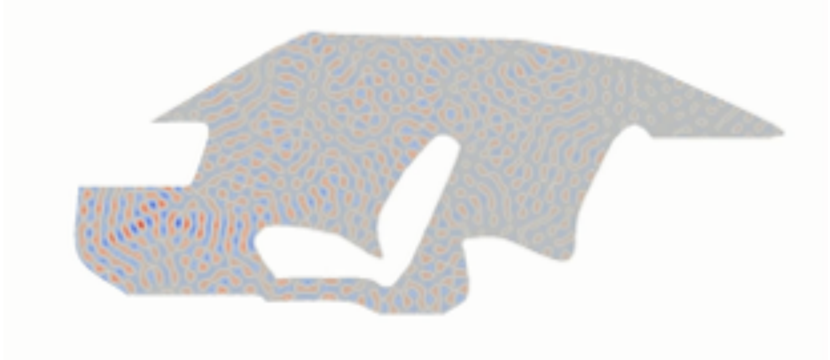
low frequency



mid-frequency



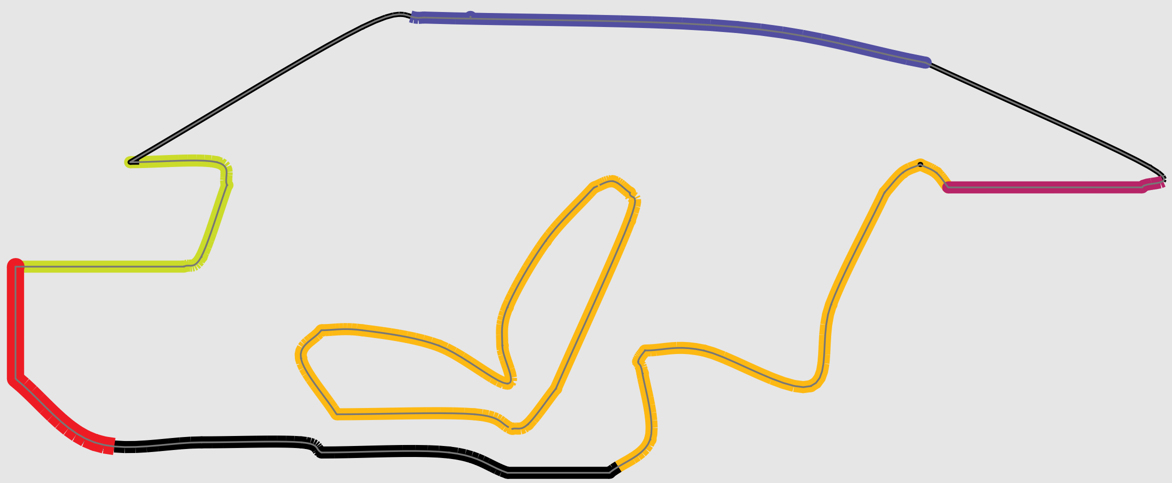
high frequency



low sensitivity to uncertainties
local quantity

high sensitivity to uncertainties
global quantity

Modelling a vibration problems



- Fixed frequency ω
- Fluid characterized by the sound speed c and density ρ

$$k = \frac{\omega}{c} \qquad v = \frac{i}{\rho_0 \omega} \underline{\nabla} p \cdot \underline{n}$$

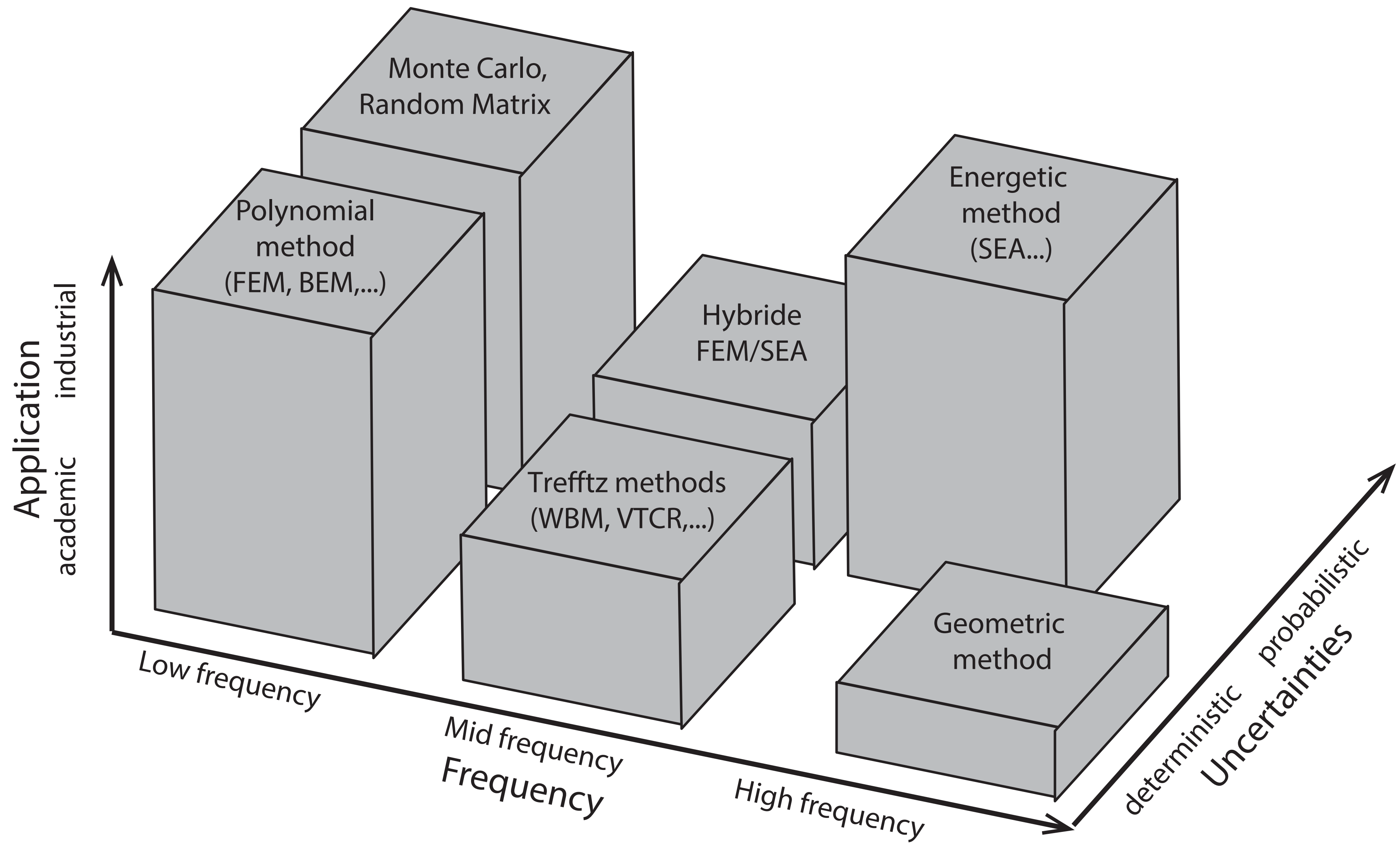
- Reference problem
  - Find $p \in H^1(\Omega)$ such that:
  - Governing equation

$$\Delta p + k^2 p = F(\underline{x}, \omega) \text{ in } \Omega$$

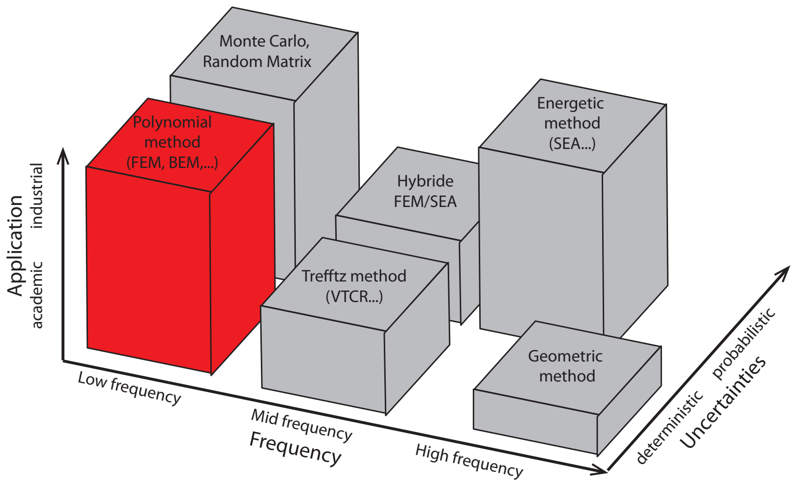
- Boundary conditions

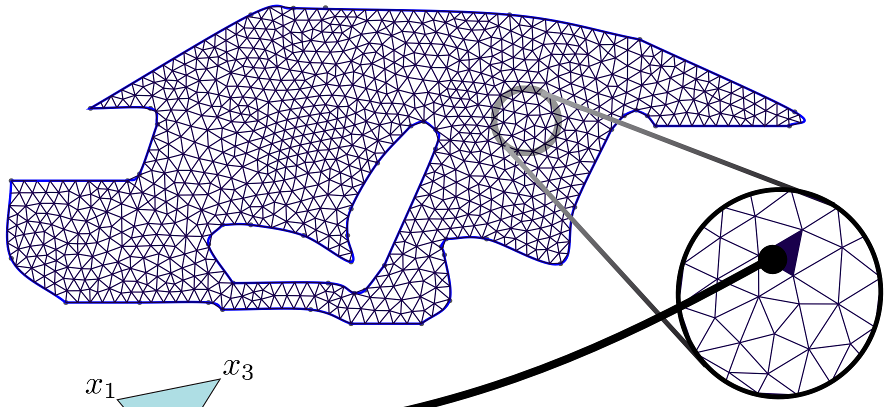
$$\left| \begin{array}{lll} p = p_d & \text{on} & \partial_p \Omega \\ v = v_d & \text{on} & \partial_v \Omega \\ p - Zv = h_d & \text{on} & \partial_Z \Omega \end{array} \right.$$

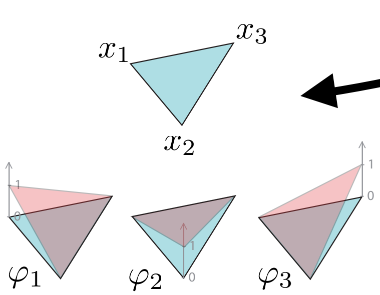
Classification of the numerical method used to solve vibration problems



The polynomial methods







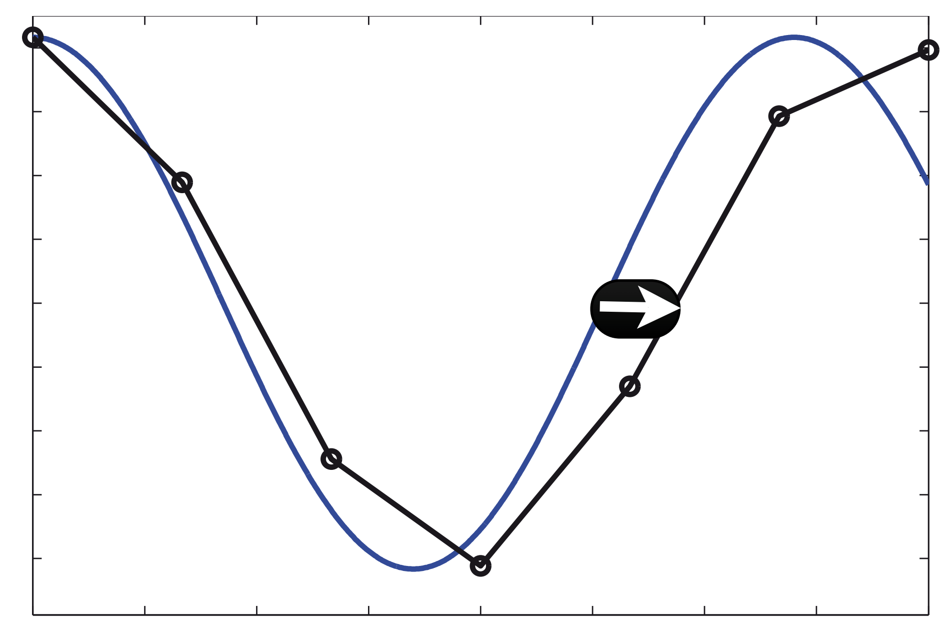
$$\alpha(\underline{X}) = \sum_i x_i \varphi_i(\underline{X})$$

Advantages: - geometrical flexibility,
- frequency independent matrices,
- sparsely populated matrices

Disadvantages: - accuracy of derivative quantities
- size model
- pollution effect

- Finite Element Method [Zienckiewicz 77]

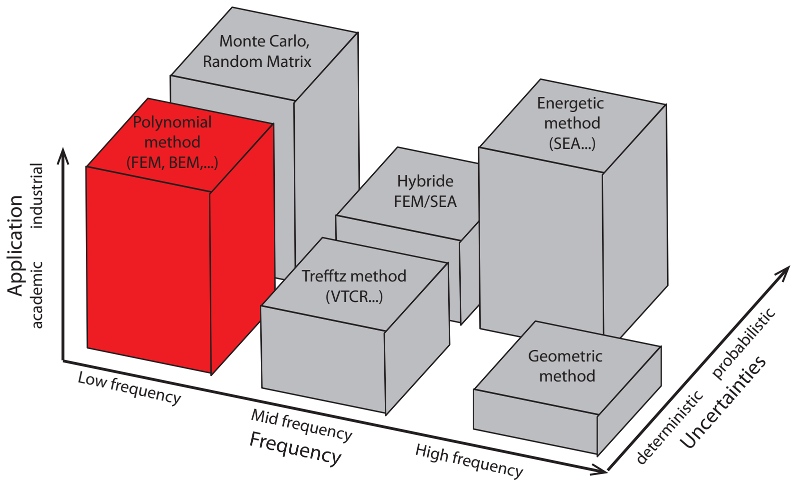
$$\varepsilon \leq C_1 \left(\frac{\omega k_a h}{p}\right)^p + C_2 \omega k_a L \left(\frac{\omega k_a h}{p}\right)^{2p}$$



Pollution error

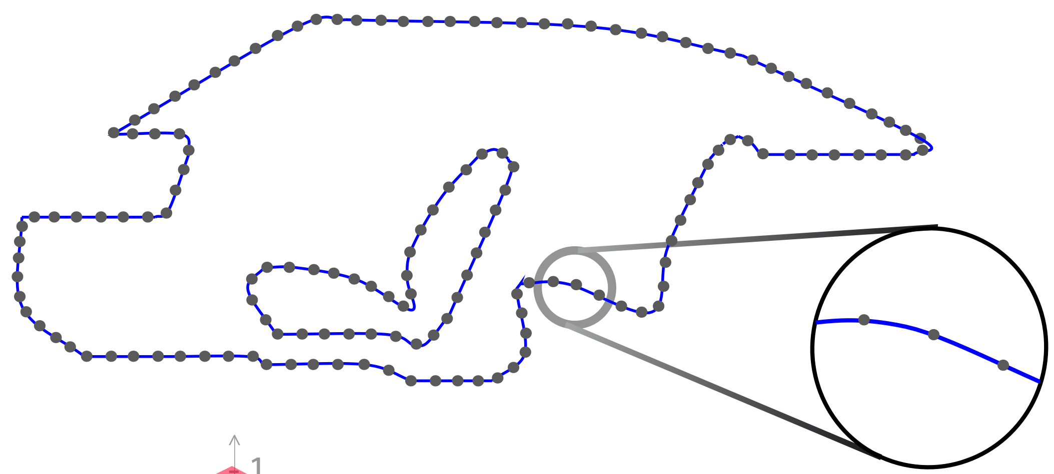
[Ihlenburg and Babuska 95, Bouillard and Ihlenburg 99]

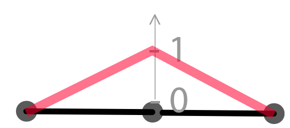
The polynomial methods



❑ Boundary Element Method

[Breddia 78, Banerjee and Butterfield 81, Bonnet 99]





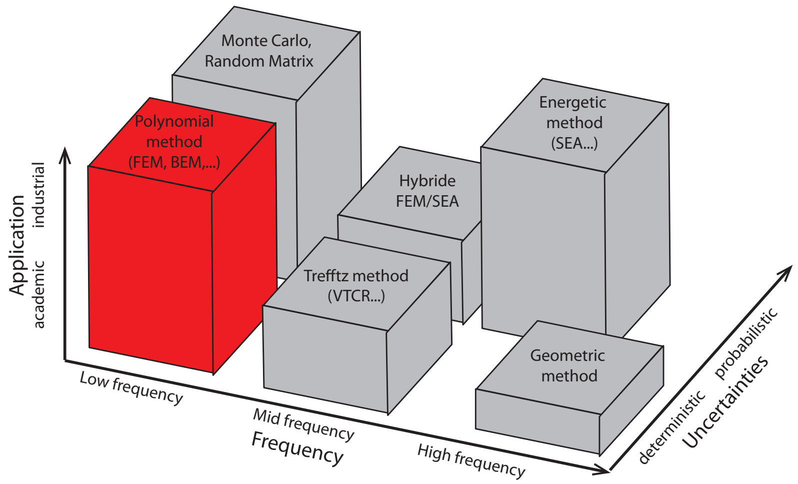
$$f(s) = \sum_i s_i \varphi_i(s)$$

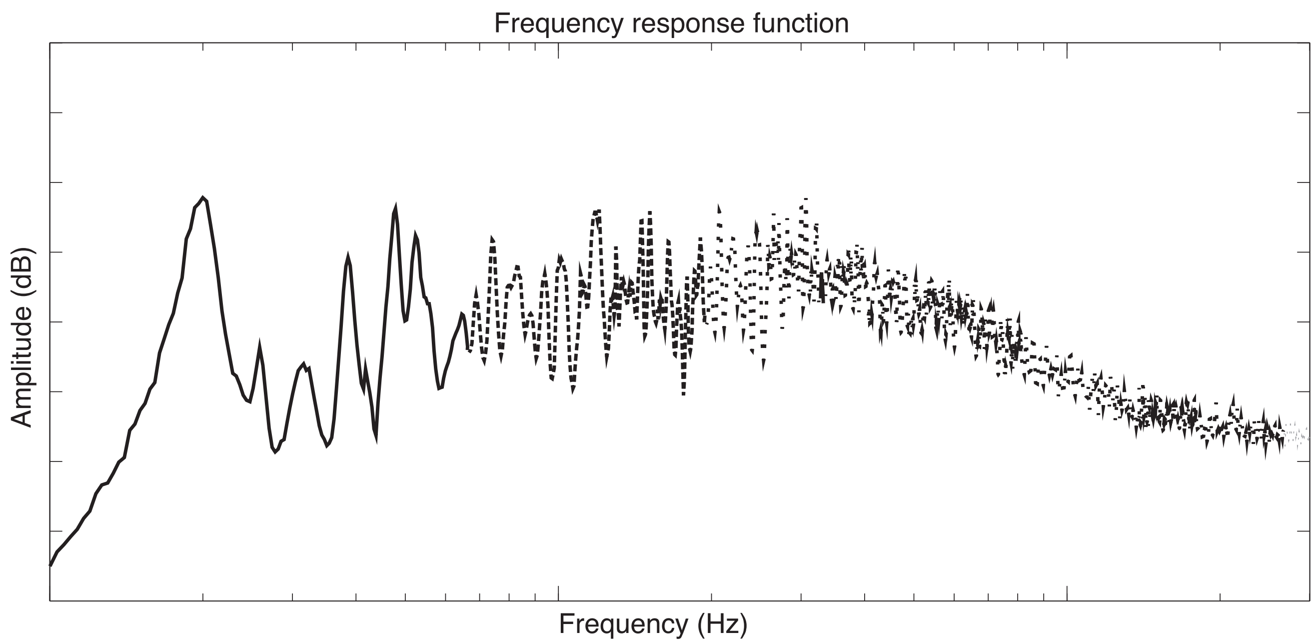
$$\alpha(\underline{X}) = \int_{\partial\Omega} G(\underline{X}, s) f(s) ds$$

Advantages: - suited for unbounded problems
- accuracy of derivative quantities
- reduced size models (compared to FEM)

Disadvantages: - complex integrations (Green functions)
- fully populated and frequency dependent matrices
- pollution effect?

The polynomial methods

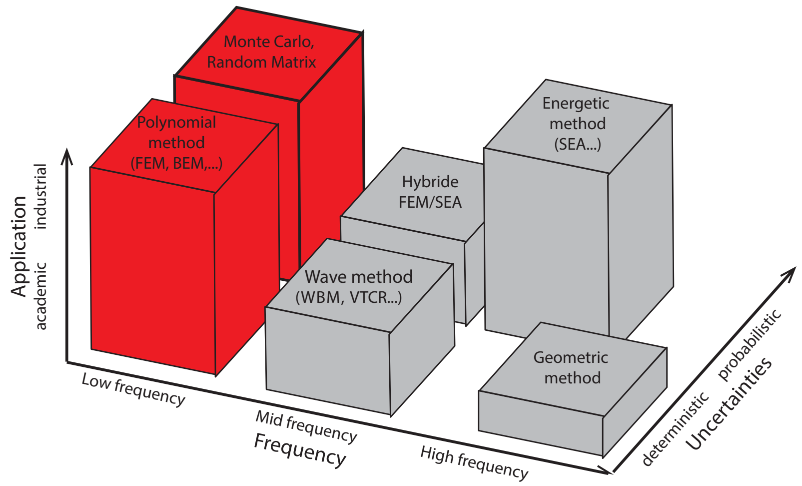




Calculation of the **response** of one **single** system

The Degrees of Freedom are the **nodal values**

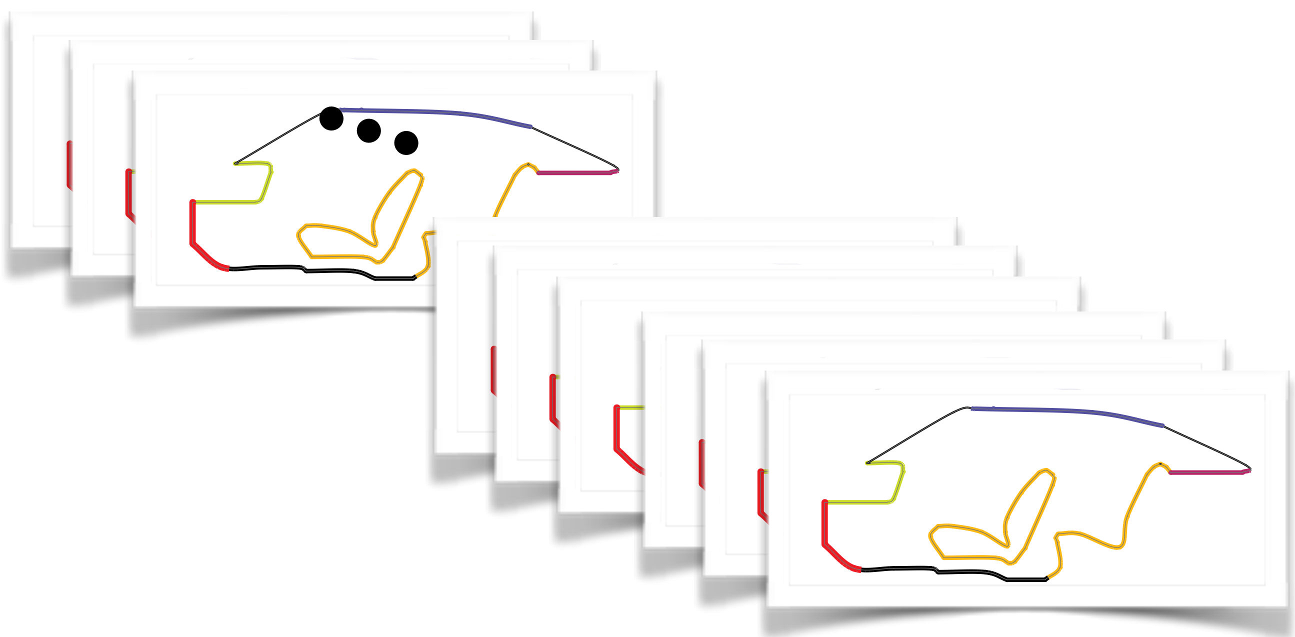
Polynomial approximation: **pollution** and **dispersion** error



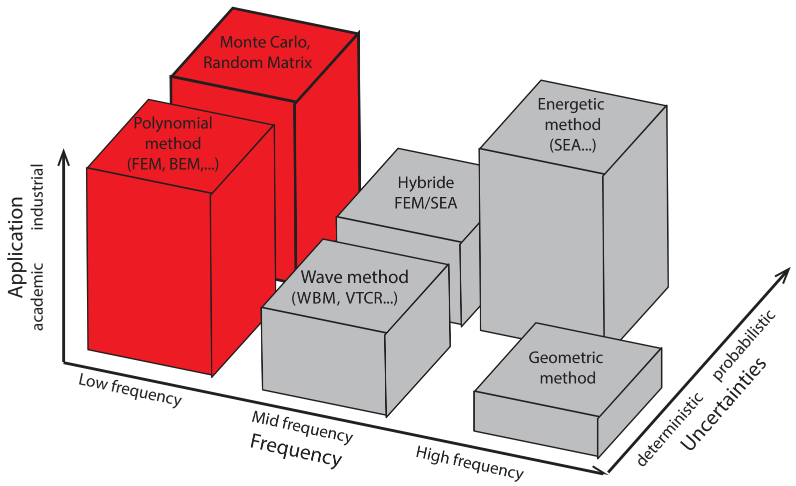
The brutal way: Monte Carlo

parametric uncertainties

Ensemble of systems



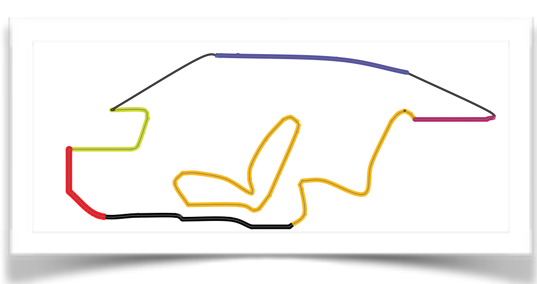
Full resolution of each member of the ensemble…



A smarter way:
Random Matrix Theory

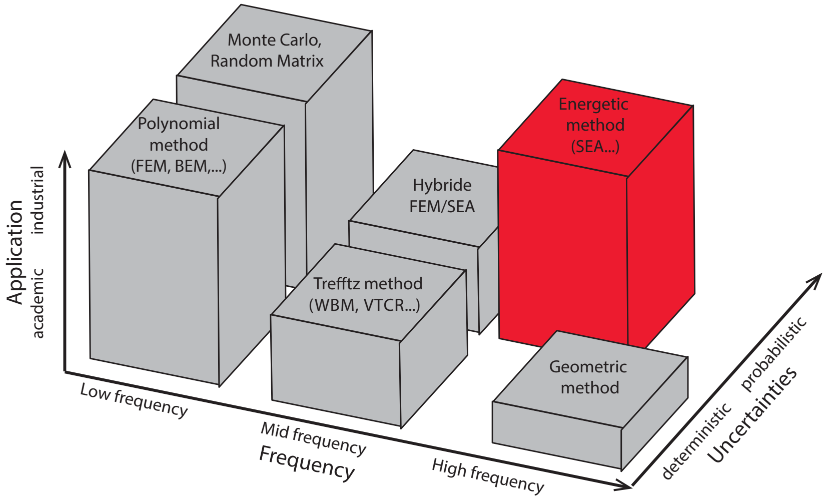
un-parametric uncertainties

Nominal system



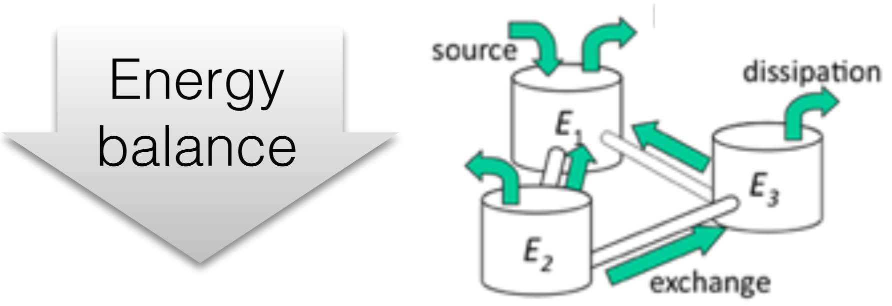
Space of random definite positive symmetric matrices, with mean correspond to FE matrices of nominal system

Statistical Energy Analysis

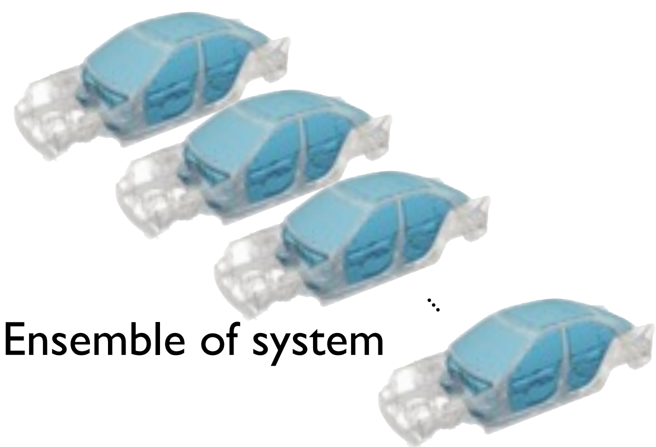


- [Lyon amd Maidanik 62]
- [Langley and Brown 04]

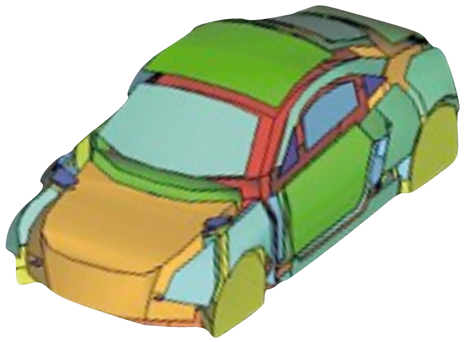
Governing equation and Boundary condition of an ensemble of system



$$\mathbf{Q}_i = \omega\eta_i\mathbf{E}_i + \sum \omega\left[\eta_{ij}\mathbf{E}_i - \eta_{ji}\mathbf{E}_j\right], \qquad i = 1, 2, \cdots, N$$



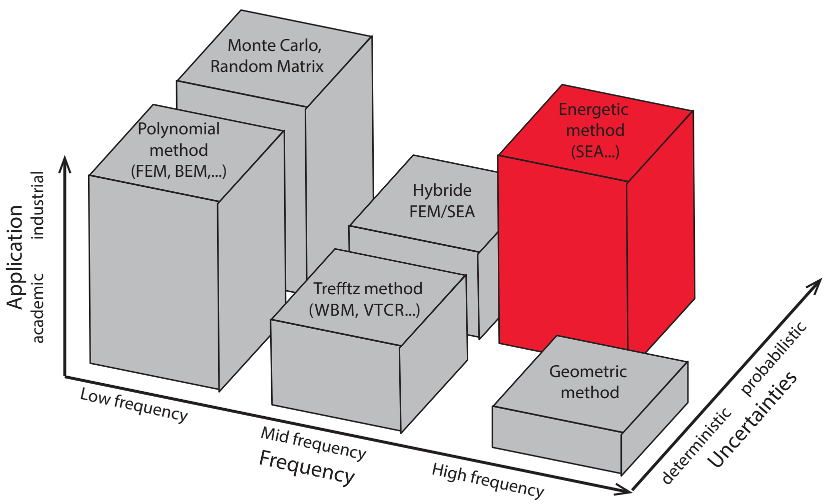
Ensemble of system



decomposition into subsystem of the nominal system

Hypothesis: - weak coupling between the subsystems (compare to losses within subsystem)
- diffuse field inside the subsystems

Statistical Energy Analysis



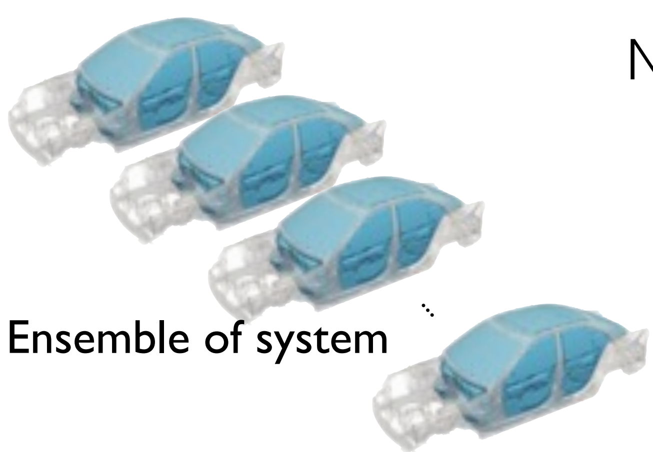
modal overlap =1

frequency

Nominal system

Variance is very small

Non-parametric
Uncertainties

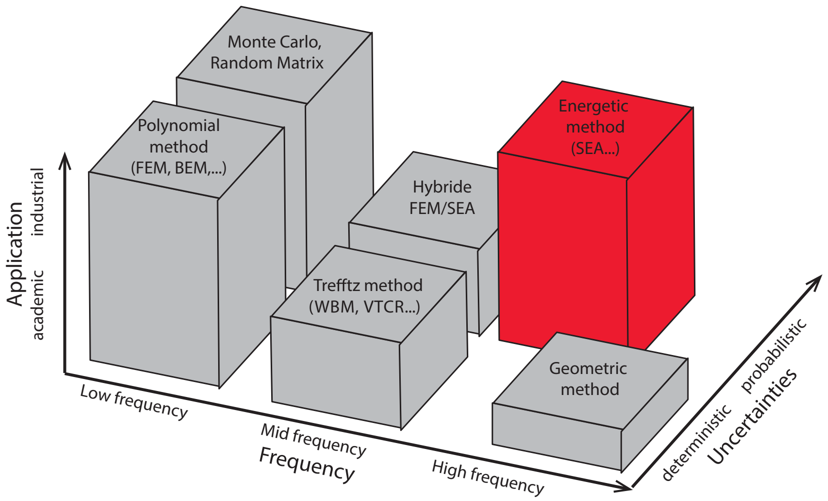


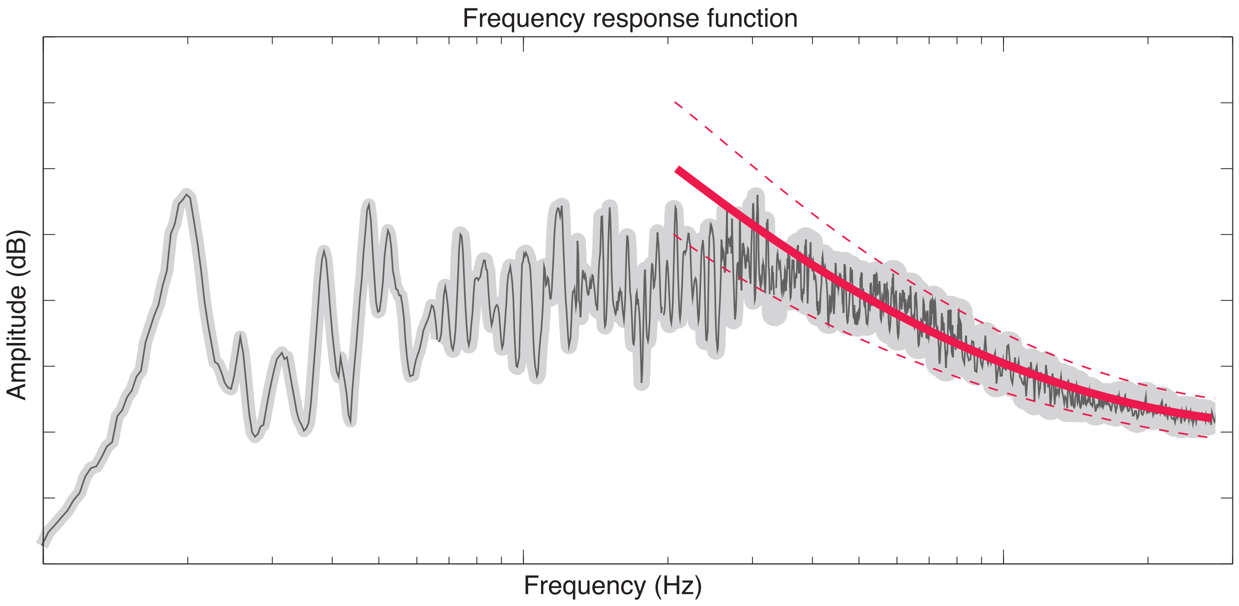
Ensemble of system

extension: SmEdA

Mean and Variance

Statistical Energy Analysis



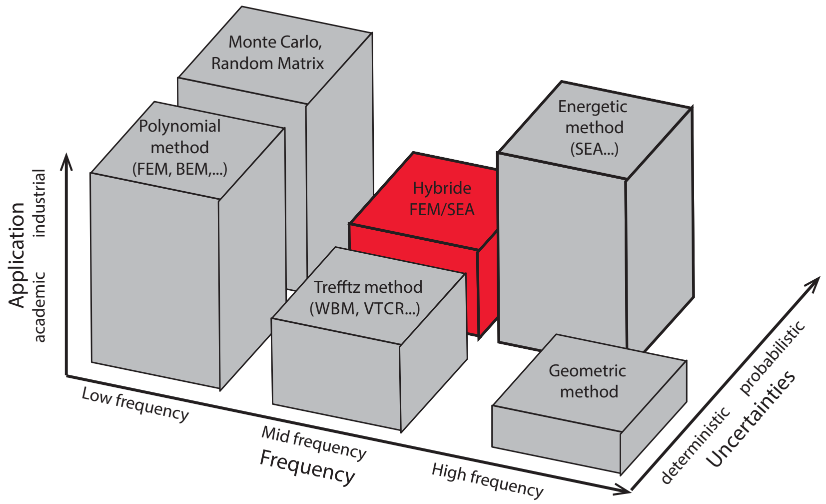


- [Lyon amd Maidanik 62]
- [Langley and Brown 04]

Estimation of the **mean value** and **variance** of the **energy** over an **ensemble of systems**

Requires: weak coupling between subsystems and presence of diffuse field within the subsystem

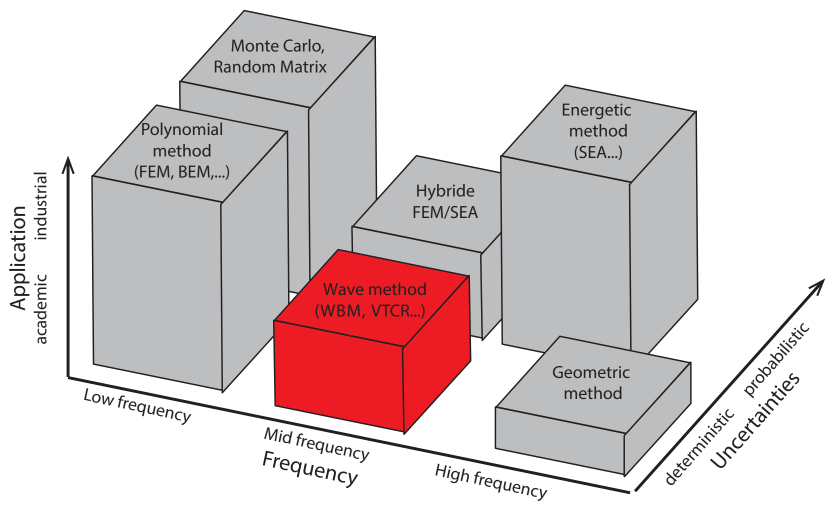
Hybrid FE-SEA



diffuse field reciprocity

FE ⟷ SEA

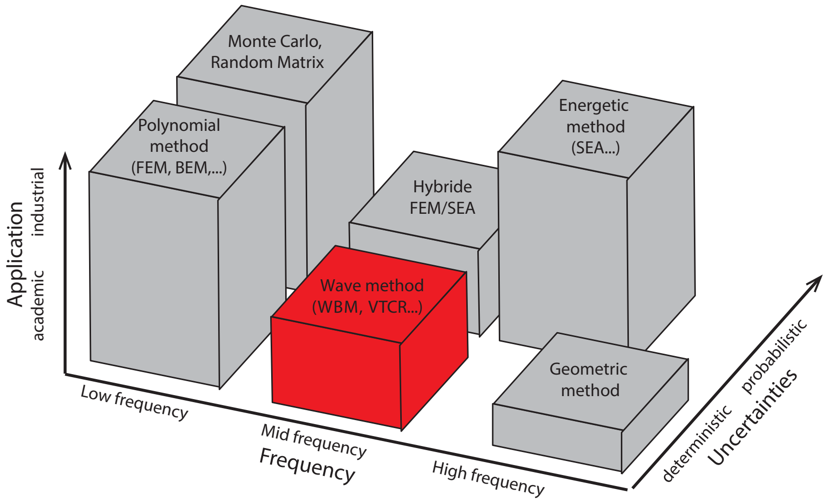
The Trefftz methods



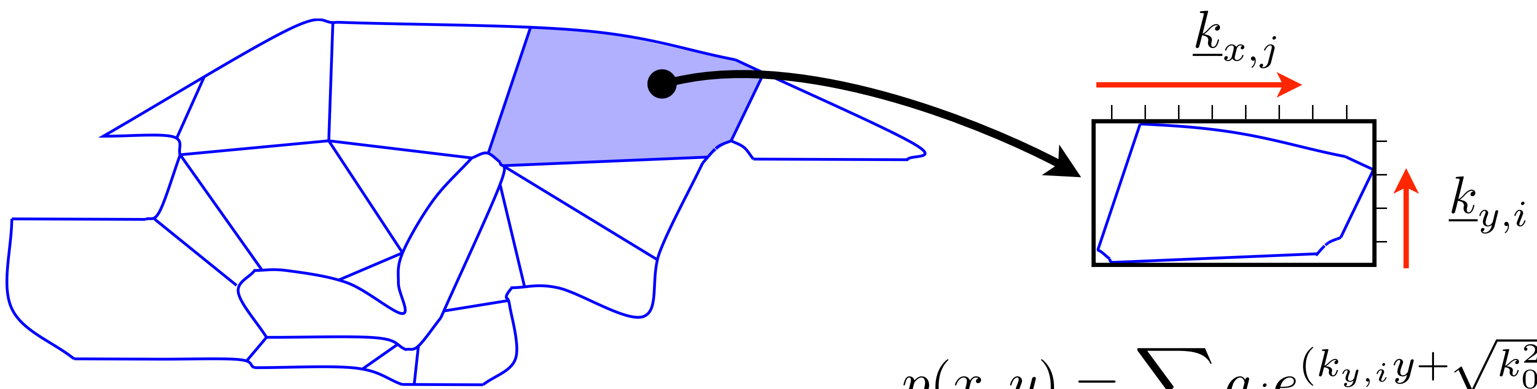
Use shape functions that are exact solution of the governing equation

WBM, VTCR, UWVF, DEM…

The Trefftz methods

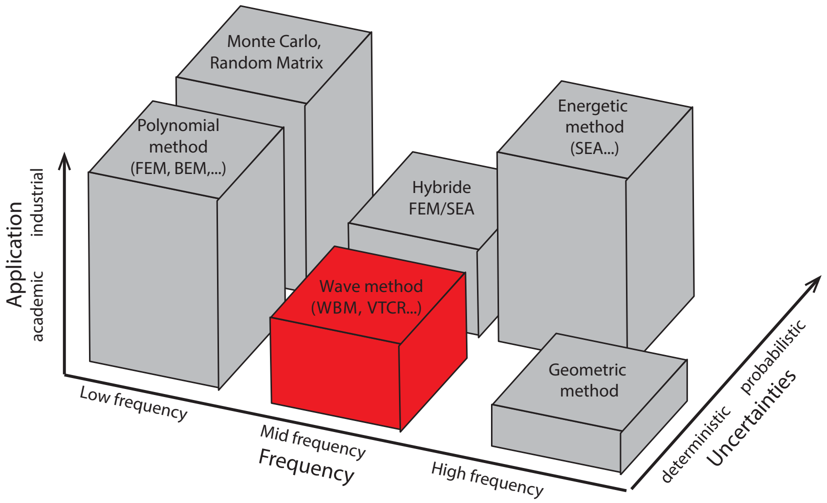


- Wave Based Method [Desmet & al. 98]

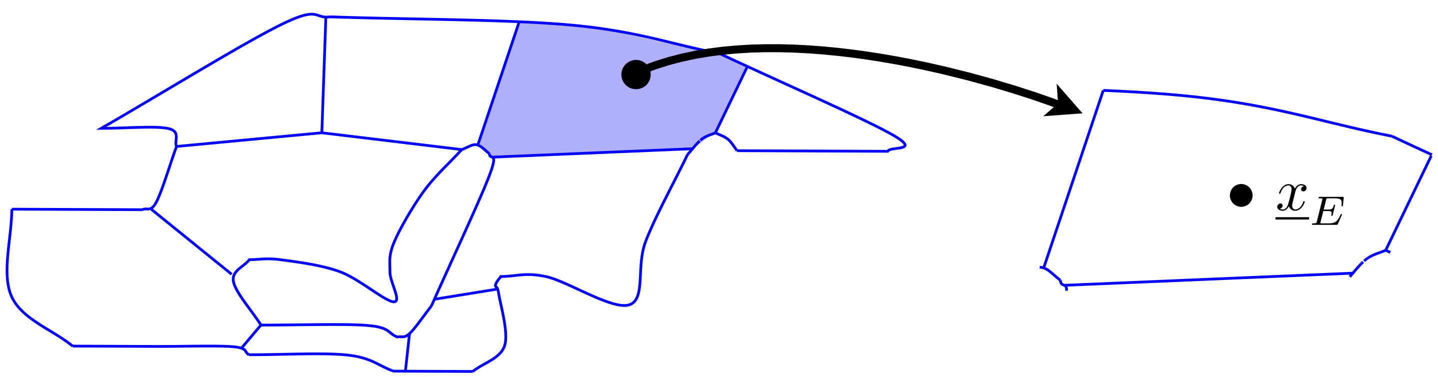


$$p(x,y) = \sum_i a_i e^{(k_{y,i}y + \sqrt{k_0^2 - k_{y,i}^2}x)} + \sum_j b_j e^{(k_{x,j}x + \sqrt{k_0^2 - k_{x,j}^2}y)}$$

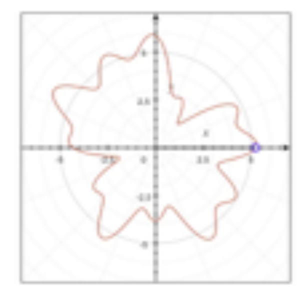
the boundary condition are enforced with weighted residual

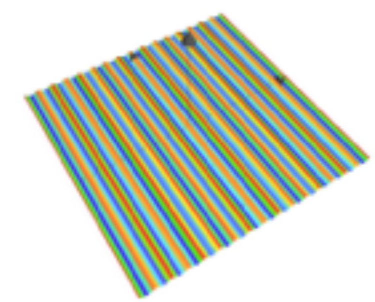


- Variational Theory of Complex Rays [Ladevèze 96]



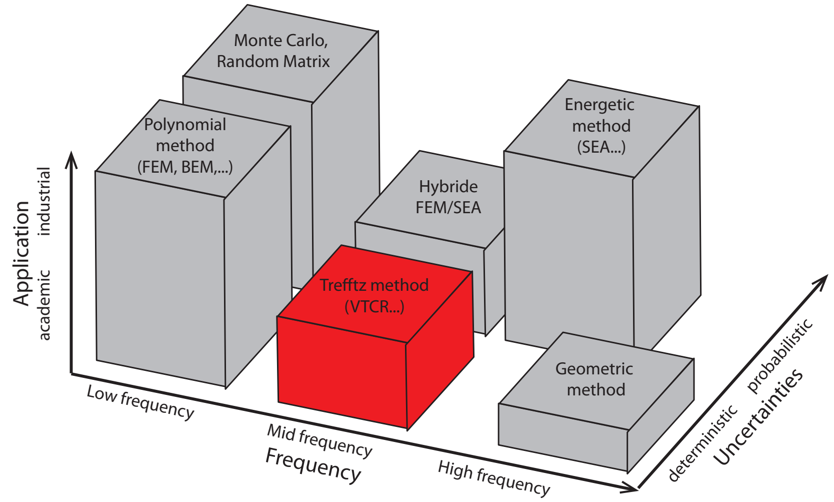
$$p_E^0(\underline{x}) = \int_0^{2\pi} A_E(\theta) e^{i\underline{k}(\theta)\cdot(\underline{x}-\underline{x}_E)} d\theta$$

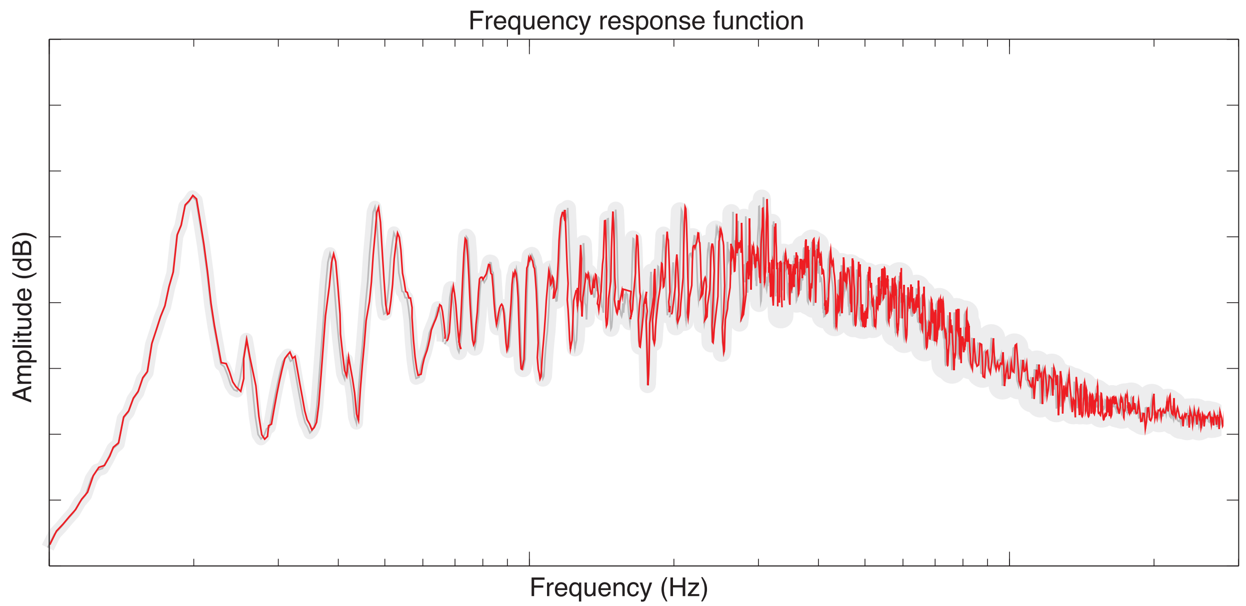




the boundary condition are enforced with a weak non symmetric formulation

The Trefftz methods



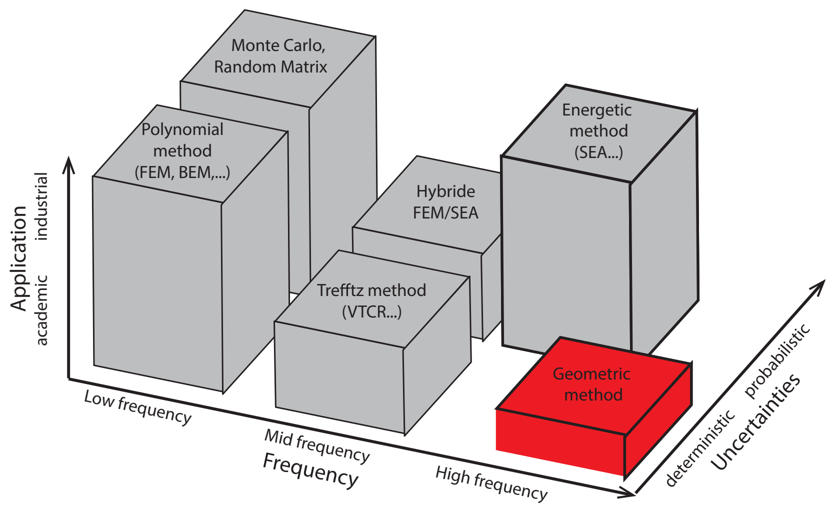


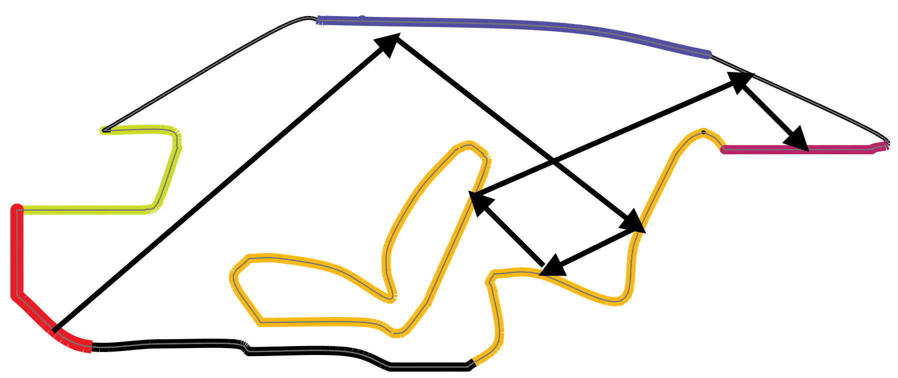
Calculation of the **response** of one **single** system

The Degrees of Freedom are the **waves amplitudes**

Could lead to hill conditioned algebraical system

Geometric method





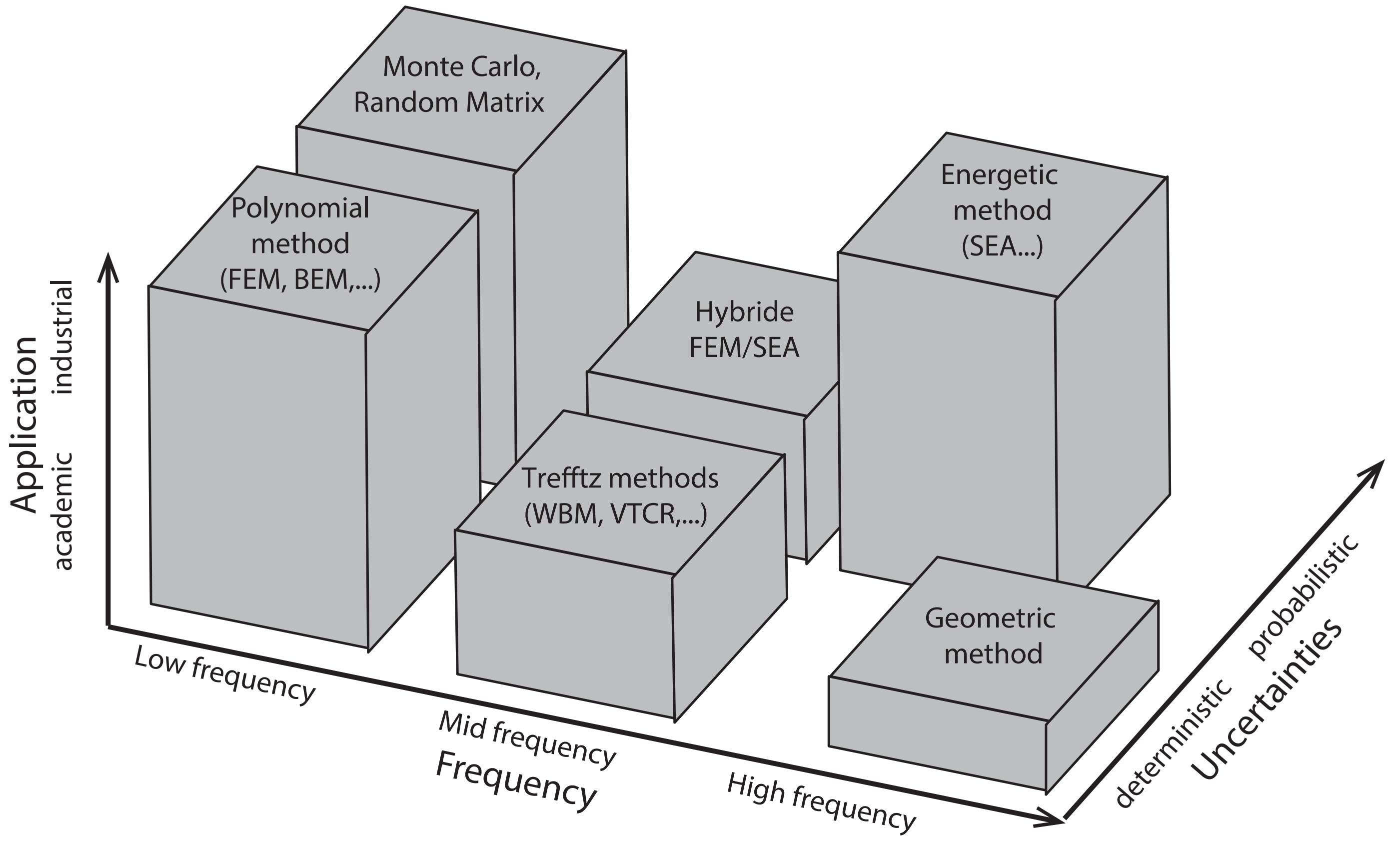
Use Rays rather than plane waves

ray tracing, radiative transfer equation, radiosity…

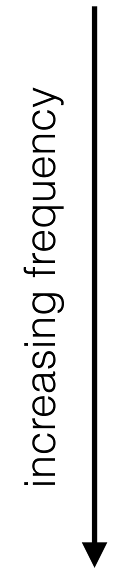
main assumption: the rays are uncorrelated

Classification of the numerical method used to solve vibration problems

There is no perfect classification…



Which quantity of interest?



- value at a specific point
- average value over a small surface
- energy inside a sub-system

Wave Finite Element, Dynamic Energy Analysis, Complex envelope?

How to validate a method?

1) run some sanity checks

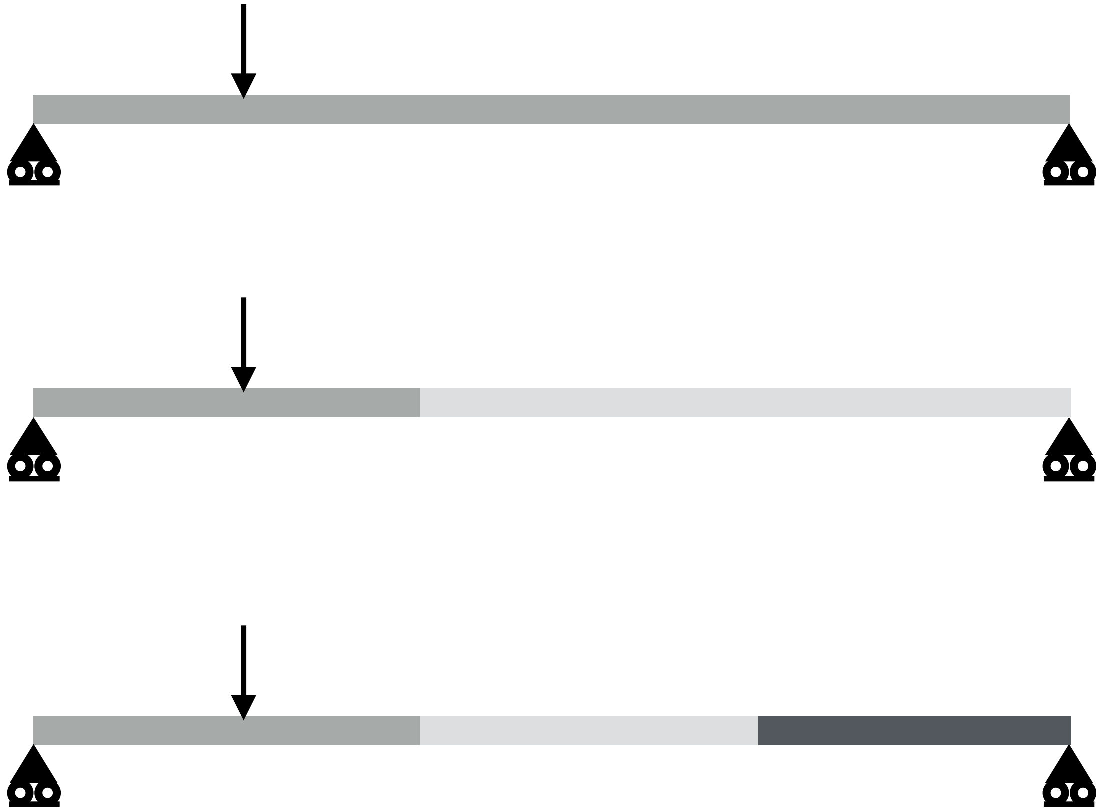
2) compare to an analytical solution (when available)

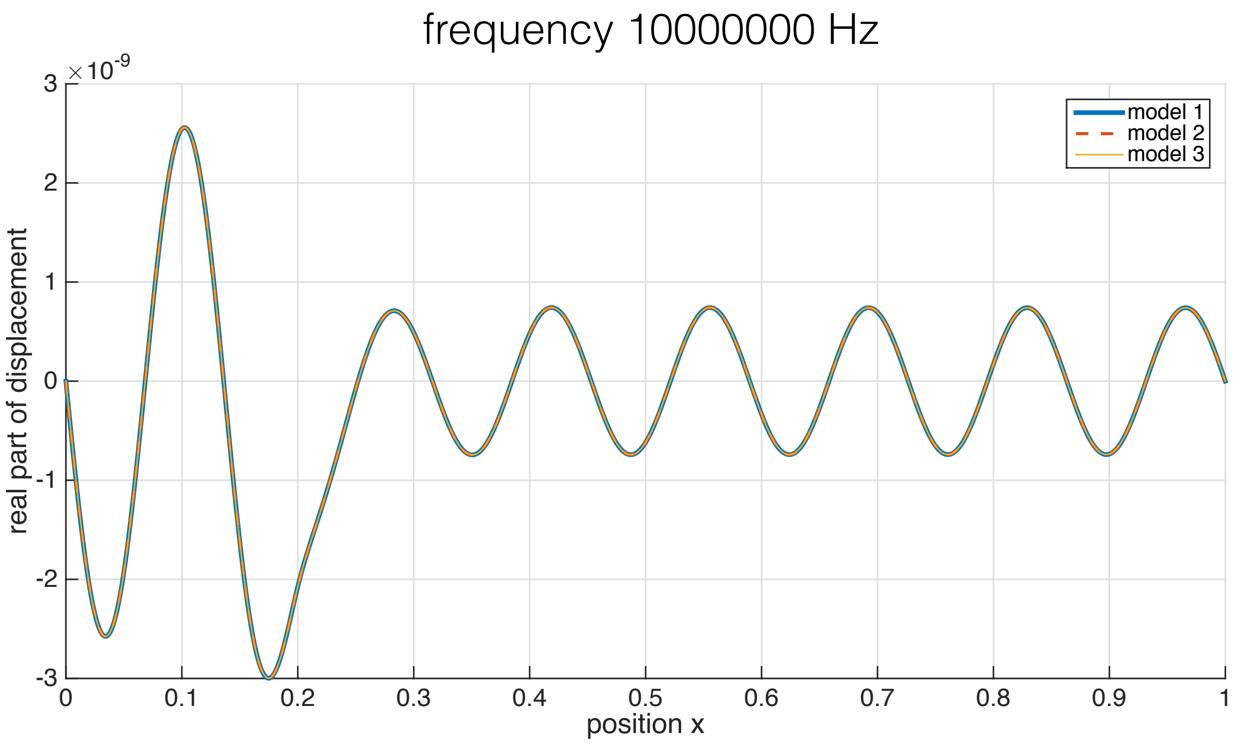
3) compare to an other numerical method

Examples of sanity check

Examples of sanity check

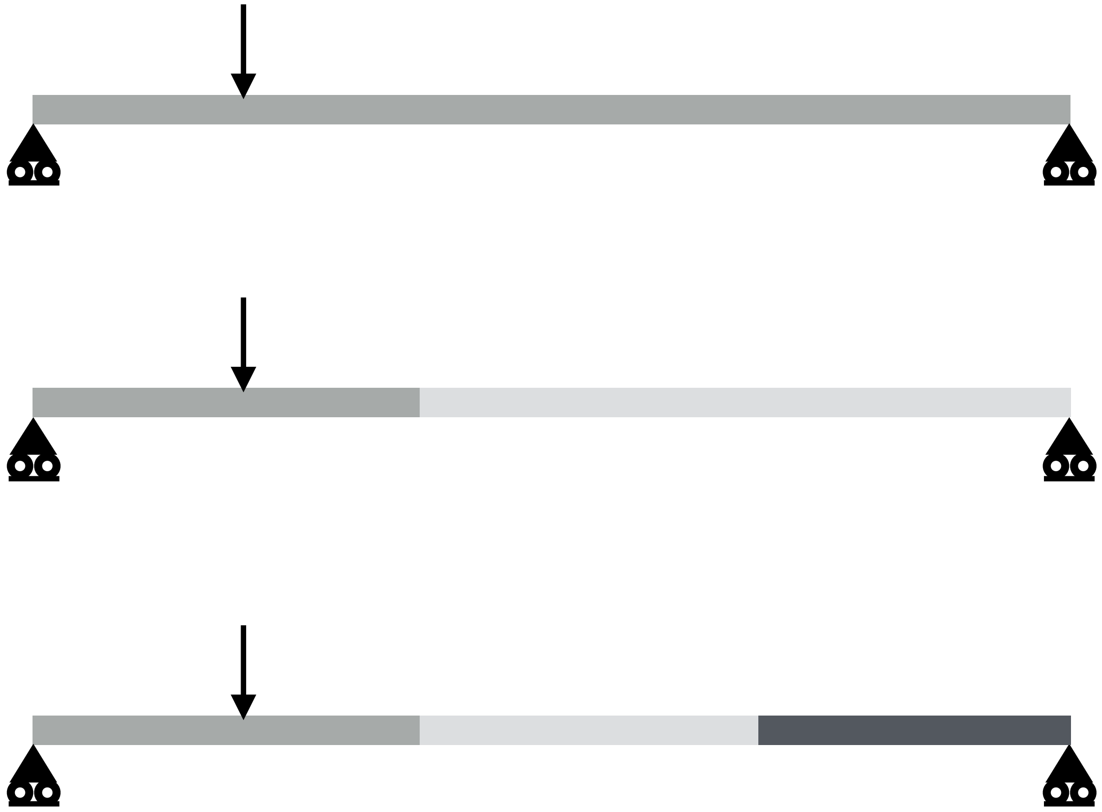
Use different models of the same problem

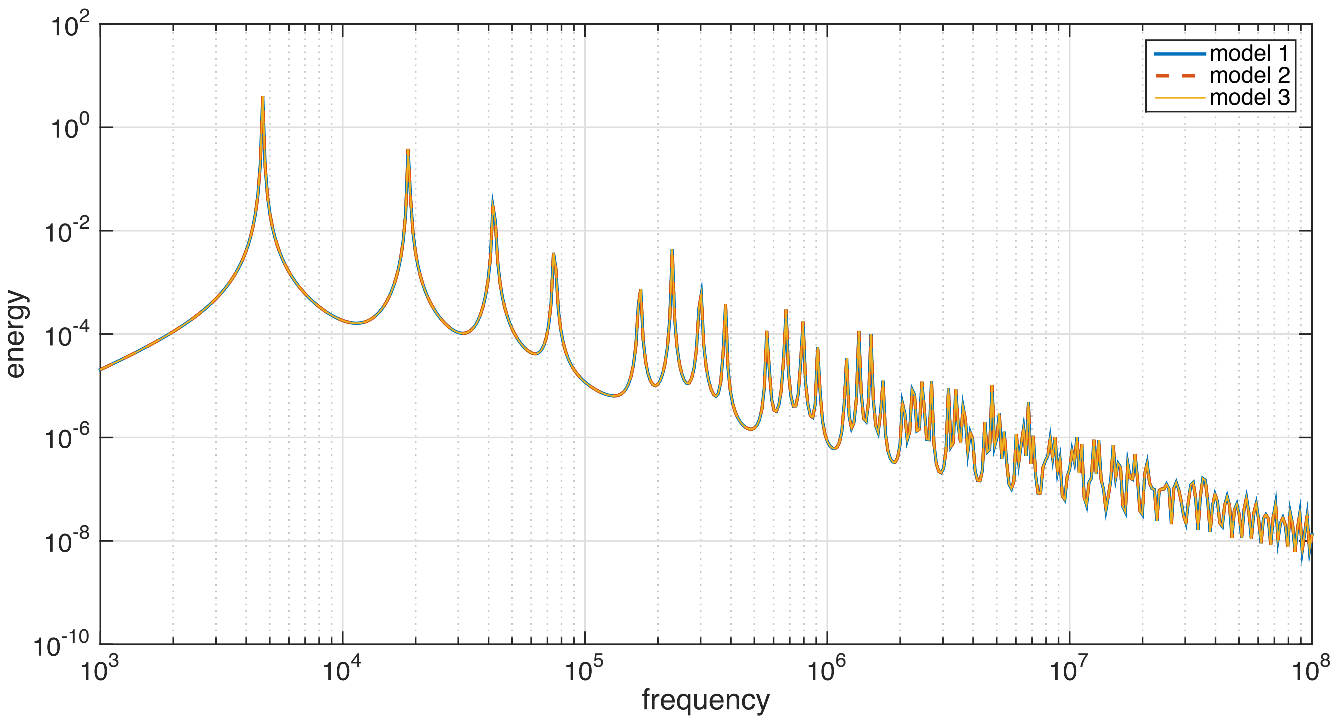




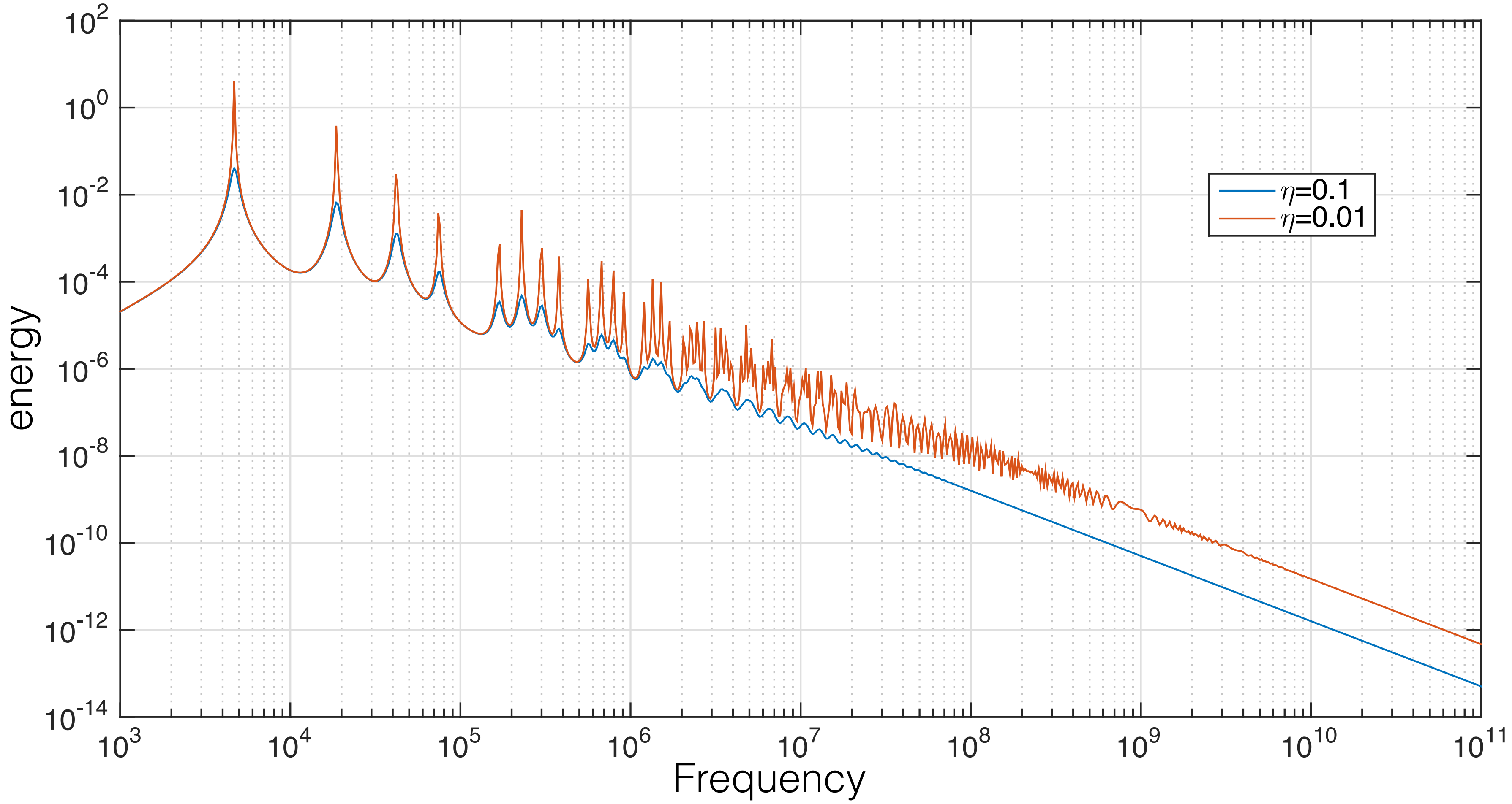
Examples of sanity check

Use different models of the same problem

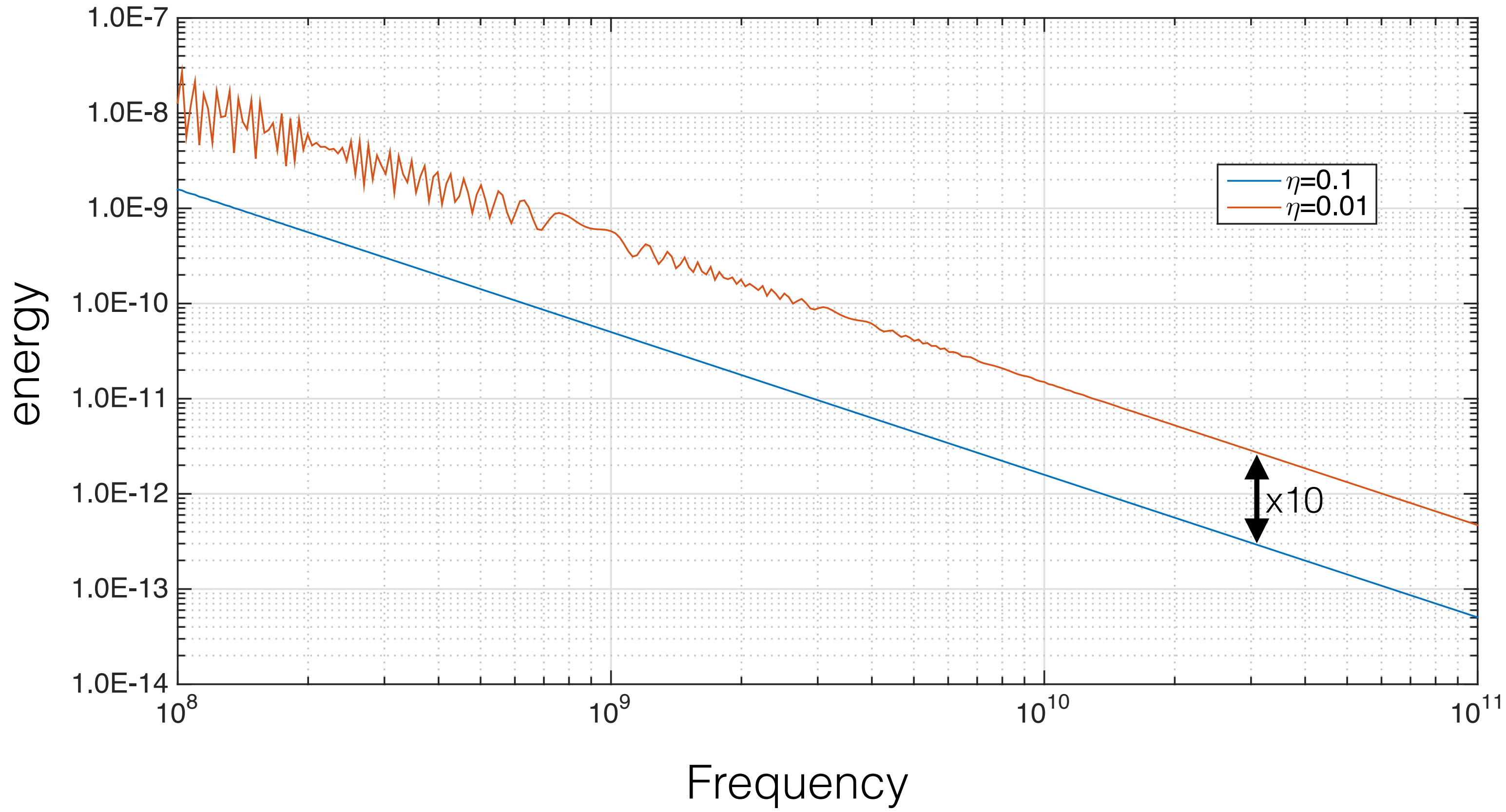




Effect of damping on the solution



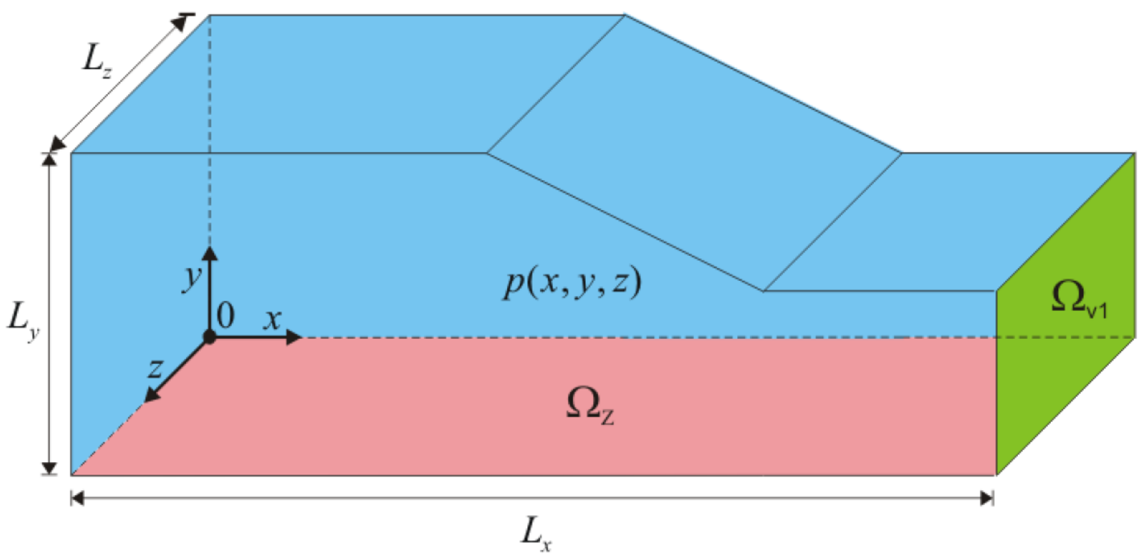
Effect of damping on the solution



Comparison with reference solution

Are you solving the same problem???

Academic Validation case 1 of Mid-Frequency Project



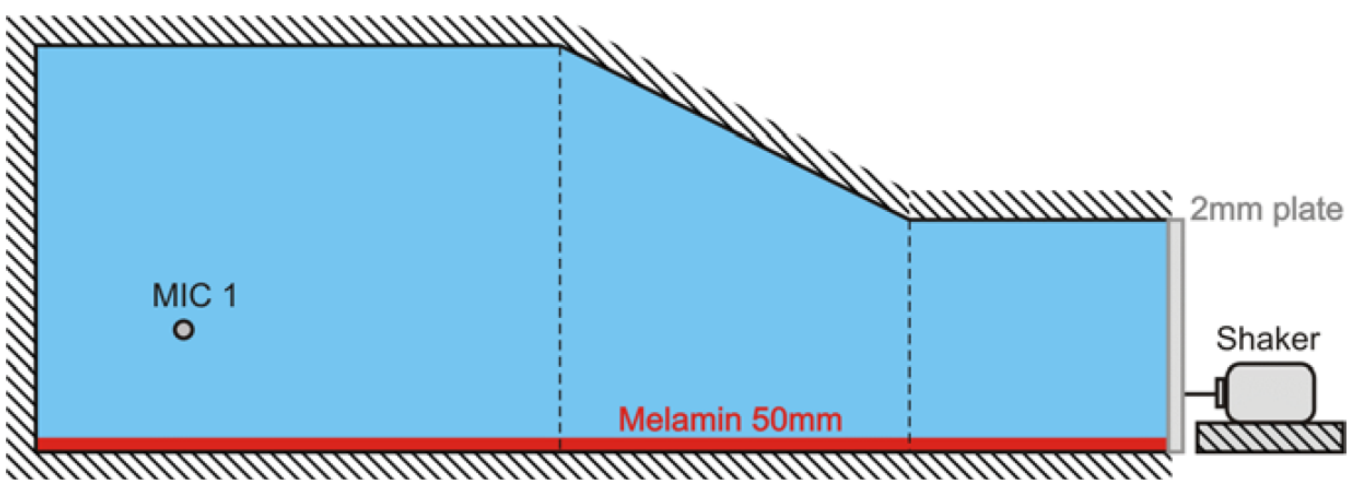
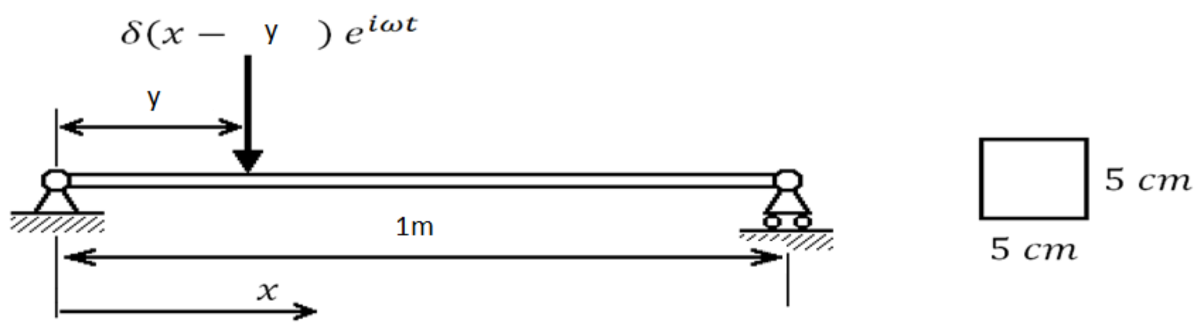


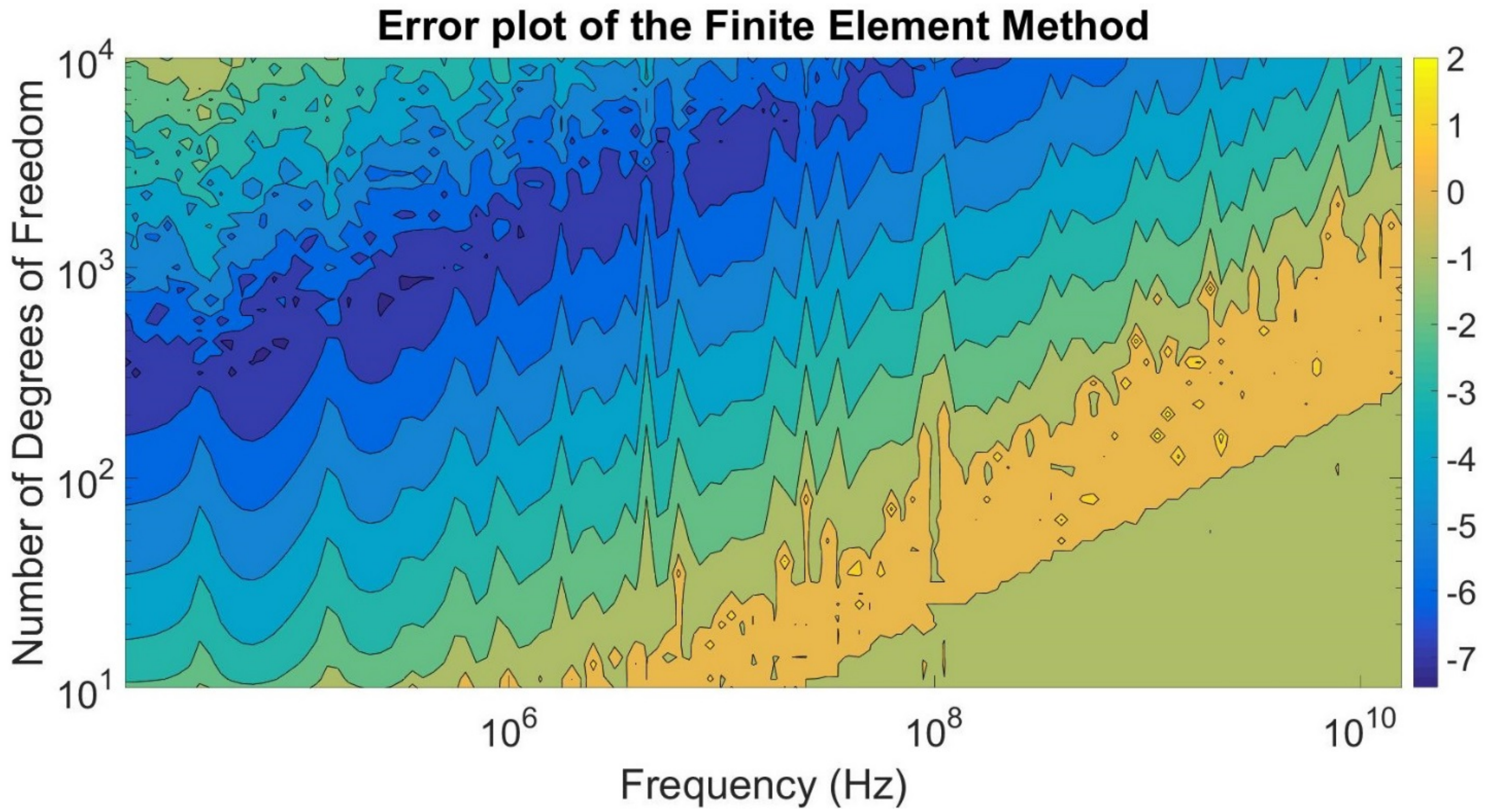
Figure 3-3: Measurements setup of car like cavity with engine bay

boundary condition, damping…

Forced harmonic vibration of a beam

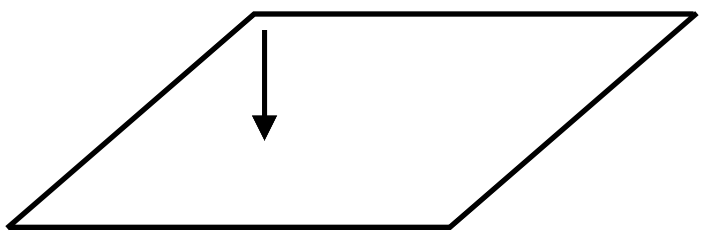


$$w^{ex}(x,\omega) = \sum_{m=1}^{\infty} \frac{2F\sin(m\pi y/L)}{L\,(E(1-i\eta)I(m\pi/L)^4 - \omega^2\rho S)}\sin(m\pi x/L)$$



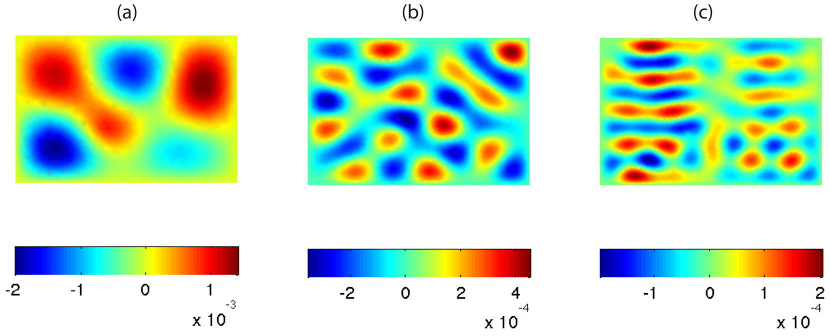
Validation against analytical solution

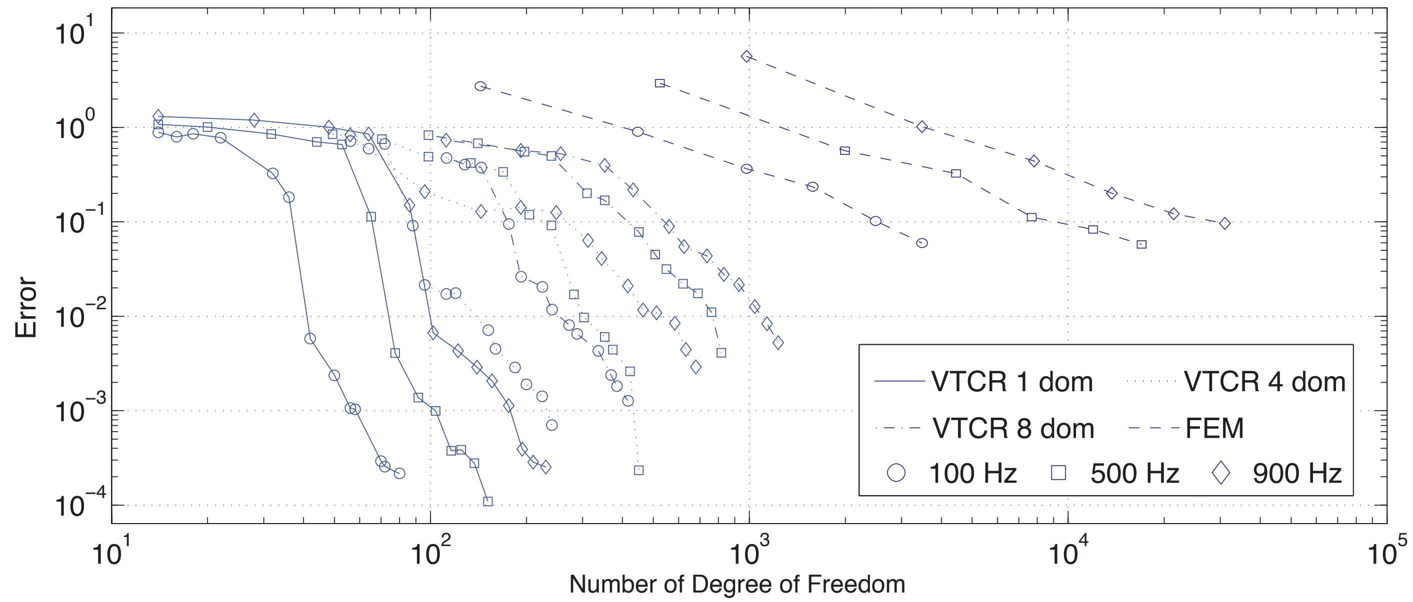
Forced harmonic vibration of a plate



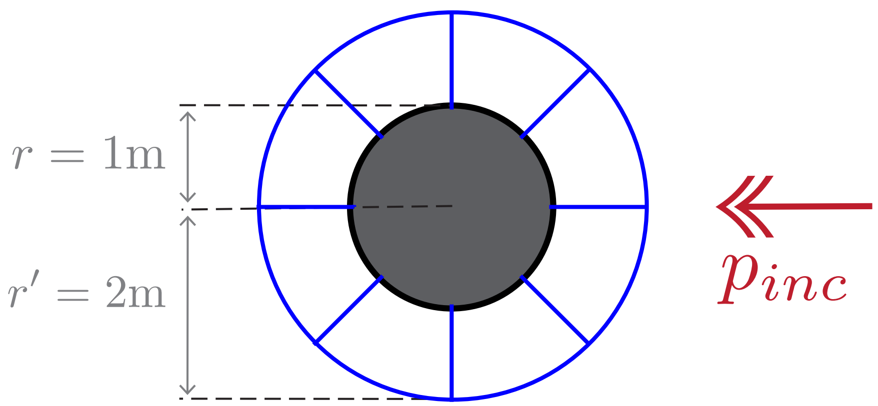
$$w^{ex}(x,y,\omega) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \frac{4F/(L_xL_y)\sin(m\pi x_F/L_x)\sin(n\pi y_F/L_y)}{\rho h\,(\omega_{mn}^2-\omega^2)}\sin(m\pi x/L_x)\sin(n\pi y/L_y)$$

with $\omega_{mn}^2 = \frac{1}{\rho h}(D_x m^4\pi^4L_x^{-4} + 2Hm^2n^2\pi^4L_x^{-2}L_y^{-2} + D_y n^4\pi^4L_y^{-4}).$



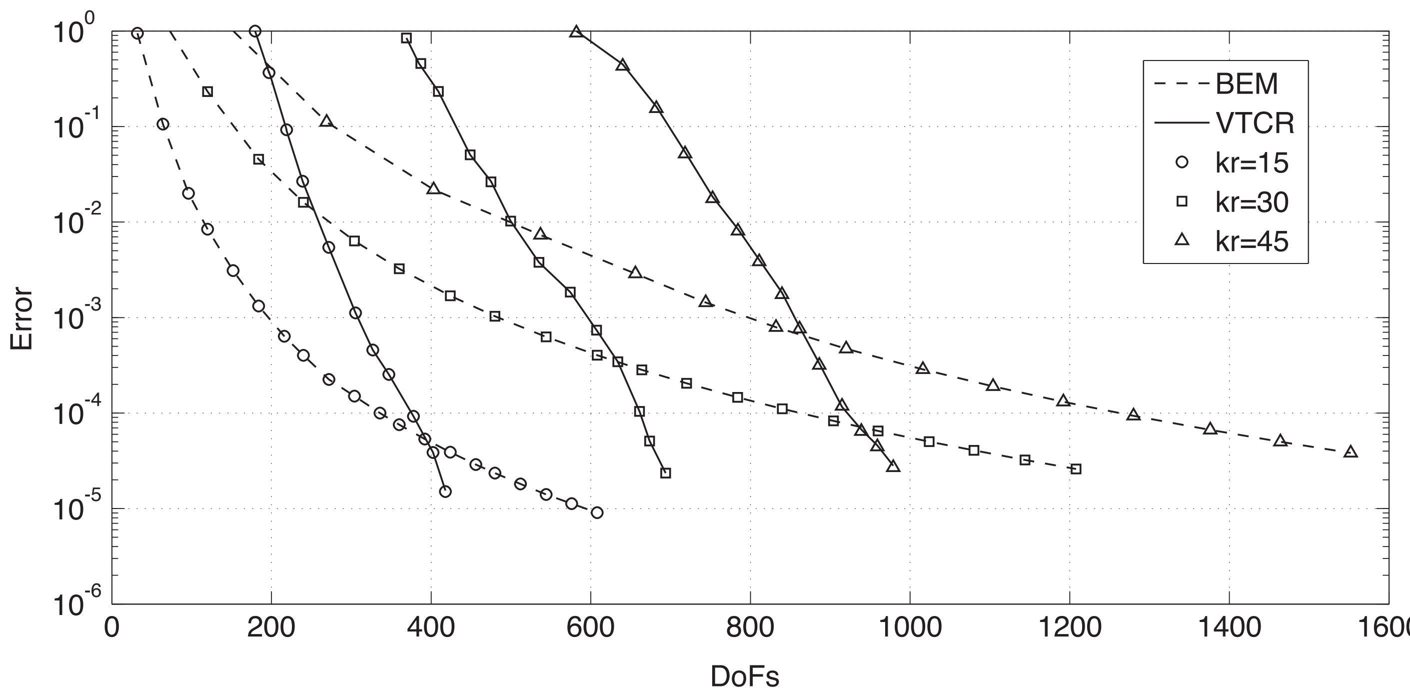


Diffraction of a plane wave by a cylinder



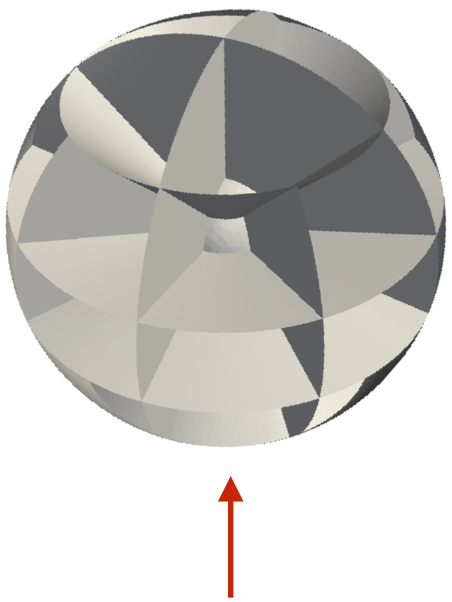
$$p^{ex}(r,\theta) = \sum_{n=0}^{\infty} a_n(\theta) H_n^{(1)}(kr) + b_n(\theta) H_n^{(2)}(kr)$$

$$\begin{bmatrix} H_n^{(1)\prime}(kR_1) & H_n^{(2)\prime}(kR_1) \\ H_n^{(1)\prime}(kR_2) - \imath k h_n^{(1)}(kR_2) & H_n^{(2)\prime}(kR_2) - \imath k H_n^{(2)}(kR_2) \end{bmatrix} \begin{bmatrix} a_n(\theta,\varphi) \\ b_n(\theta,\varphi) \end{bmatrix} = \begin{bmatrix} -(2n+1)\imath^n k P_n(cos\theta) J_n'(kR_1) \\ 0 \end{bmatrix}$$



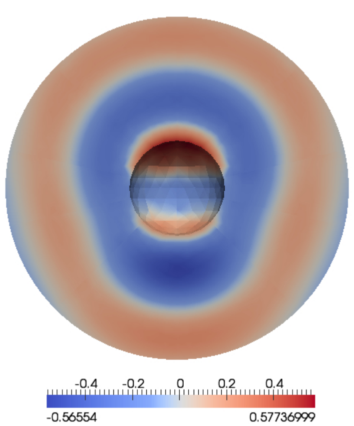
Validation against analytical solution

Diffraction of a plane wave by a sphere

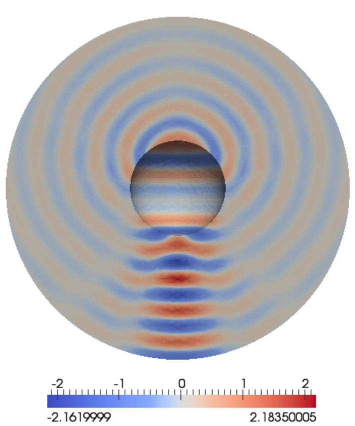


$$p^{ex}(r,\theta,\varphi) = \sum_{n=0}^{\infty} a_n(\theta,\varphi) h_n^{(1)}(kr) + b_n(\theta,\varphi) h_n^{(2)}(kr)$$

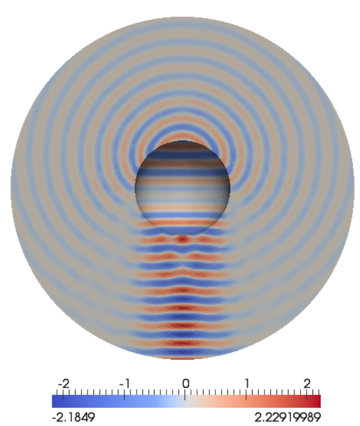
$$\begin{bmatrix} h_n^{(1)\prime}(kR_1) & h_n^{(2)\prime}(kR_1) \\ h_n^{(1)\prime}(kR_2) - \imath k h_n^{(1)}(kR_2) & h_n^{(2)\prime}(kR_2) - \imath k h_n^{(2)}(kR_2) \end{bmatrix} \begin{bmatrix} a_n(\theta,\varphi) \\ b_n(\theta,\varphi) \end{bmatrix} = \begin{bmatrix} -(2n+1)\imath^n k P_n(cos\theta) j_n'(kR_1) \\ 0 \end{bmatrix}$$



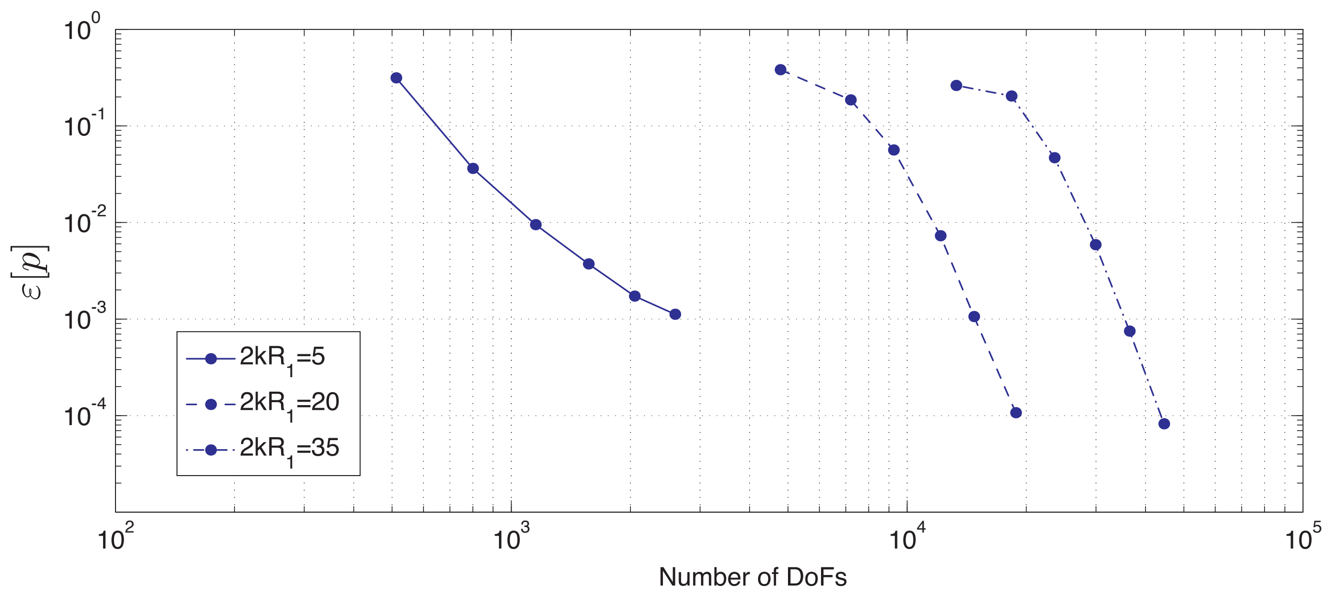
Real part of the pressure (Pa)



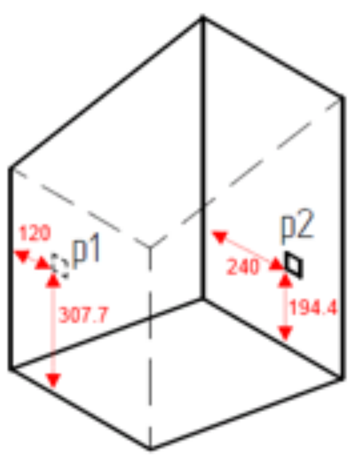
Real part of the pressure (Pa)



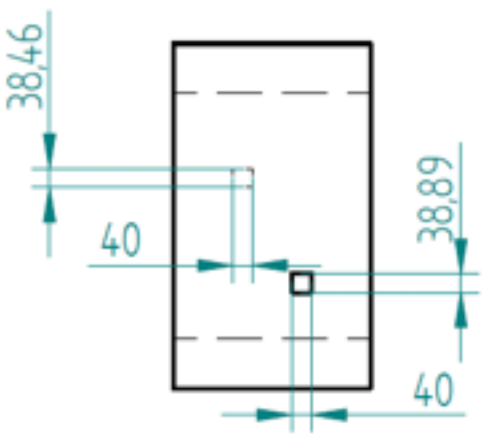
Real part of the pressure (Pa)

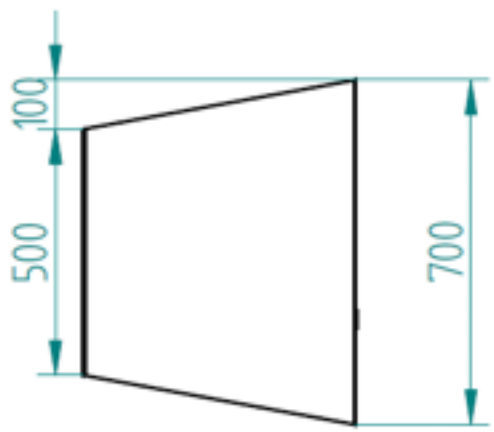


Wave method and FE



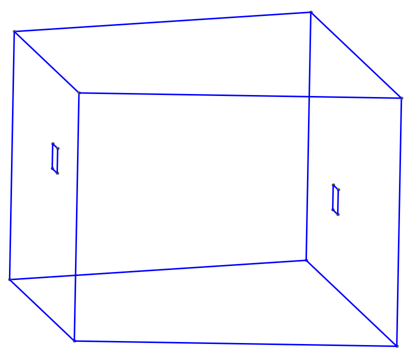
Frequency range: 10 to 2000Hz, frequency step 1Hz,
Material properties: rho = 1.2kg/m3, c=340m/s, damping ratio = 0.01).
Box with rigid wall.
The patch named p1 is the source (unit velocity) and patch p2 is the receiver.
The computed impedance is then the averaged pressure on patch p2 for a unit velocity of patch p1.



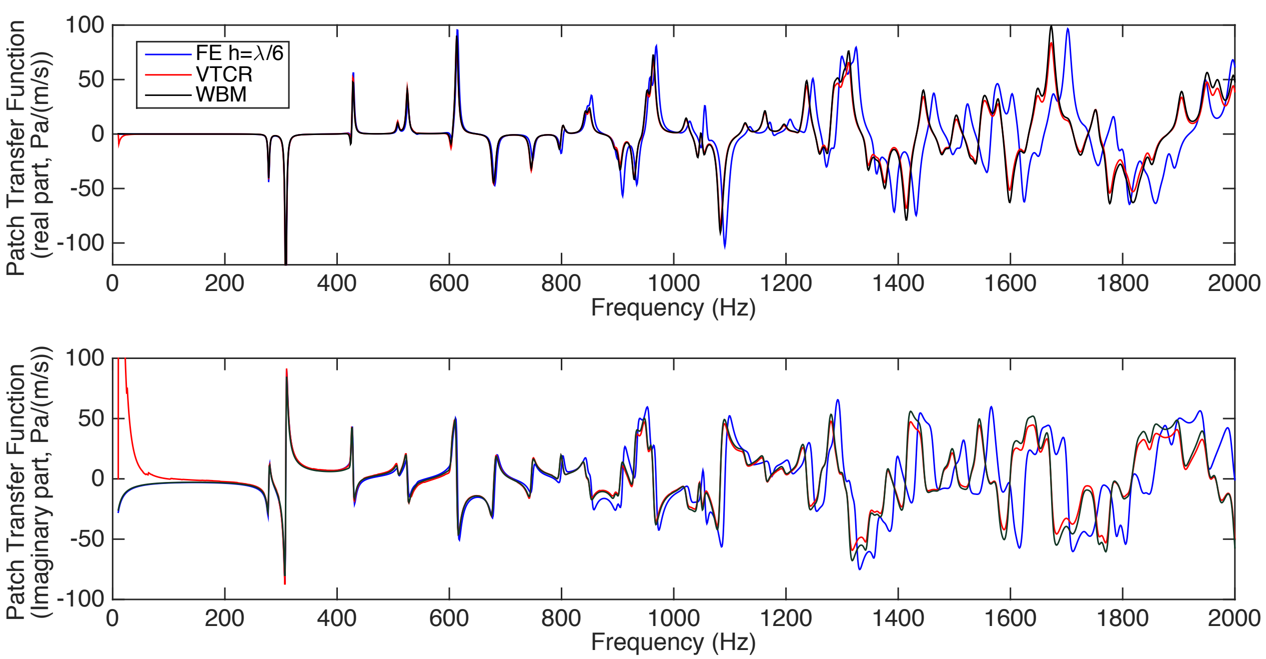


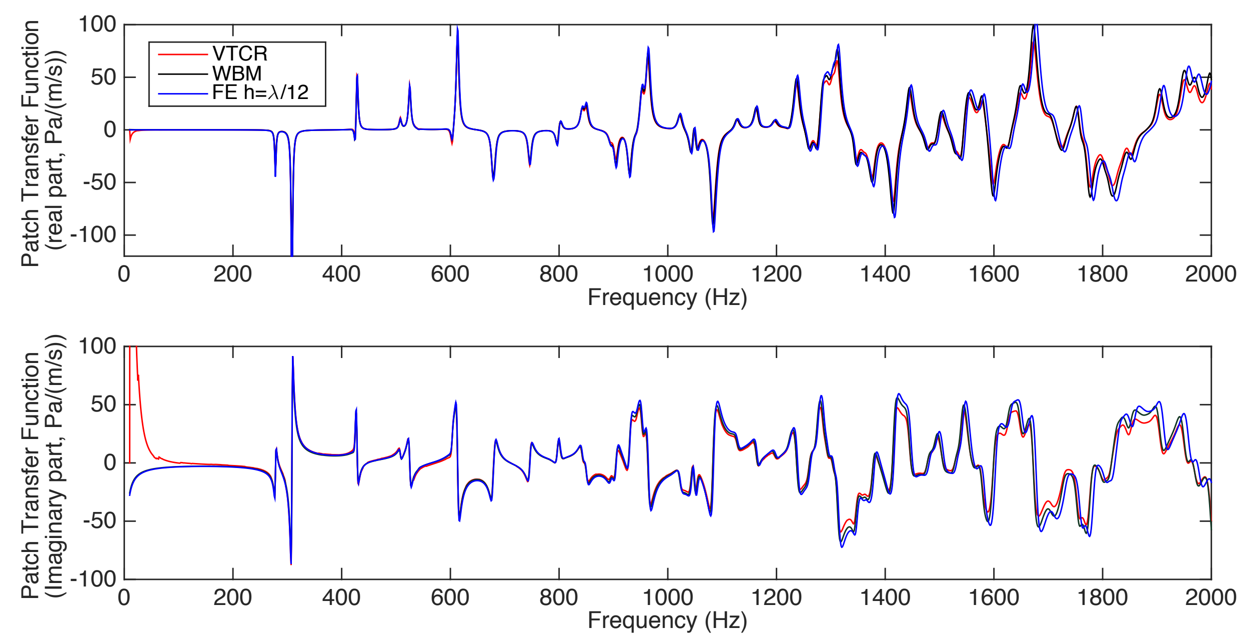
Validation against numerical solution

Wave method and FE



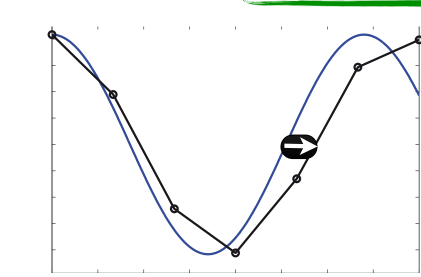
transfer function between the two patches



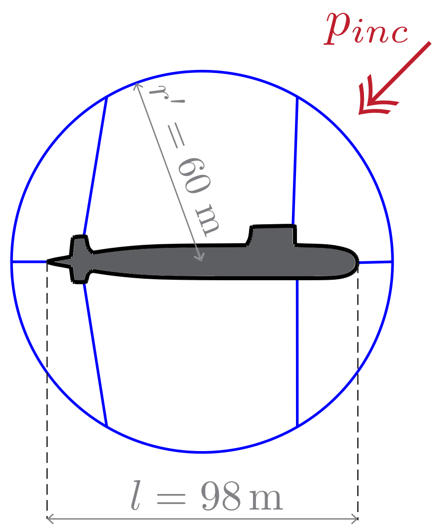


when using FE as reference, make sure the pollution and dispersion error is controlled !

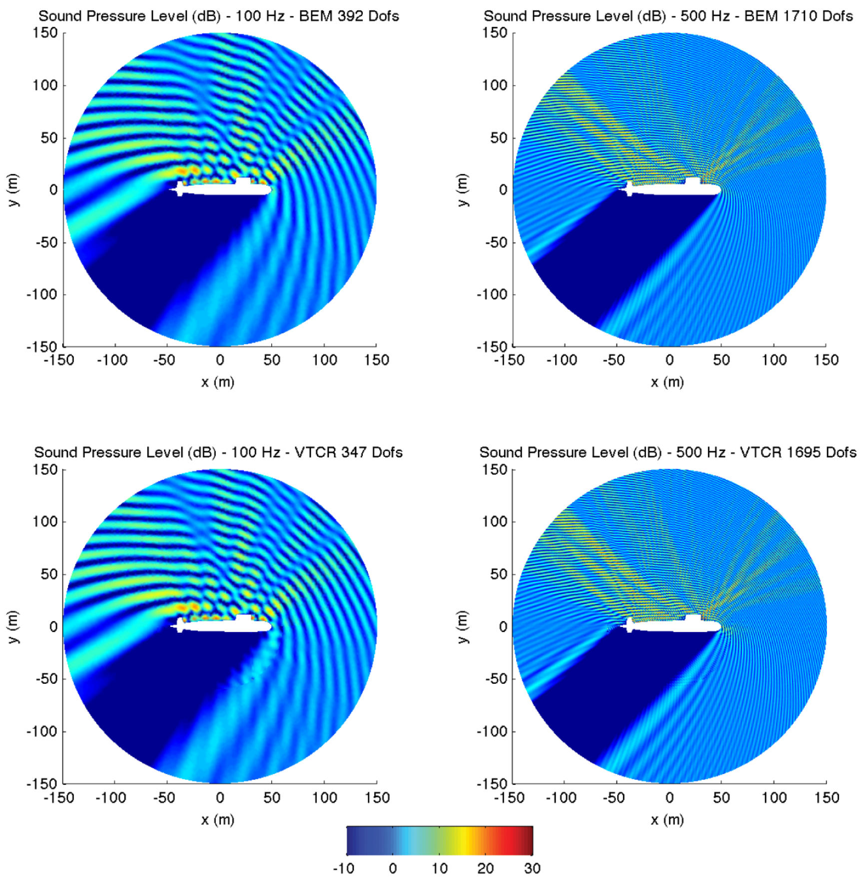
$$\varepsilon \le C_1 \left(\frac{\omega h}{c\,p}\right)^p + C_2 \frac{\omega}{c} L \left(\frac{\omega h}{c\,p}\right)^{2p}$$

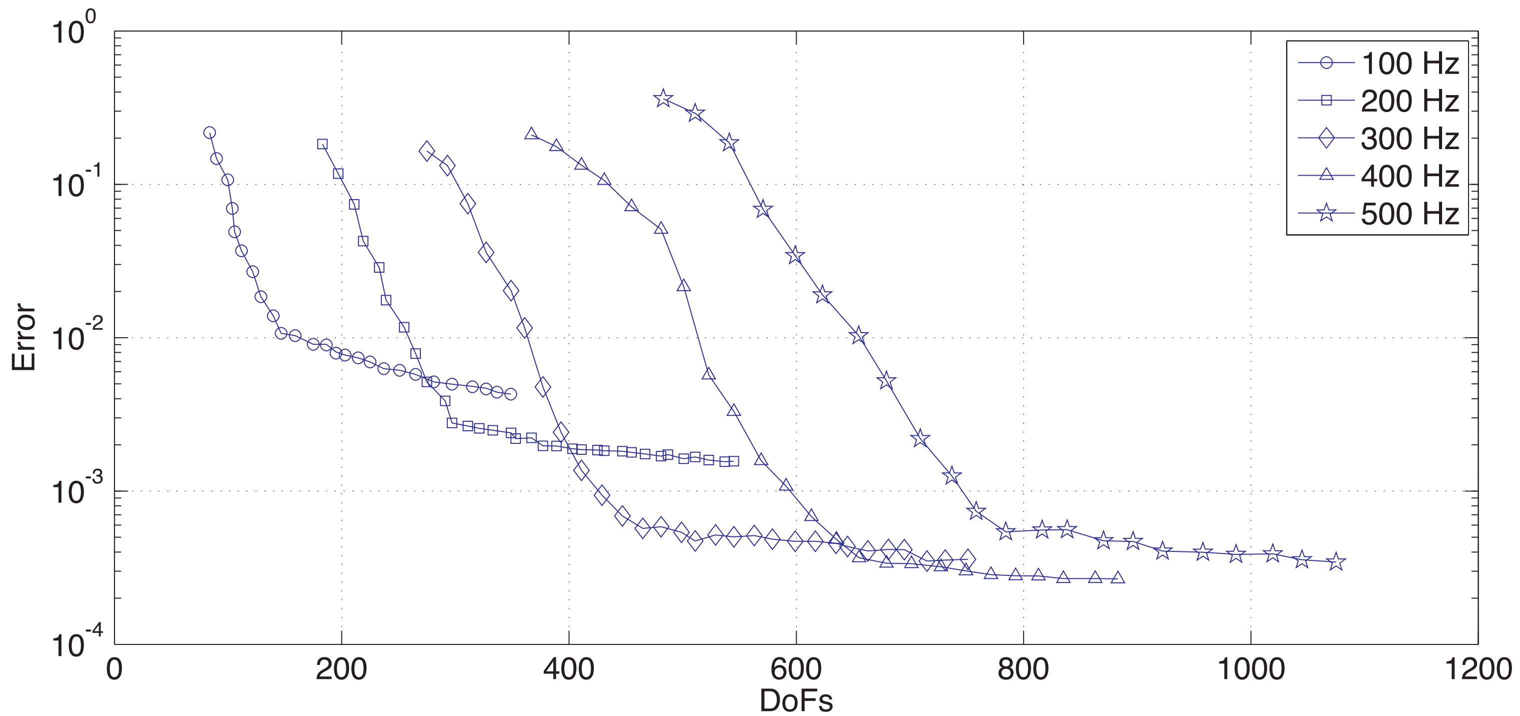


Trefftz method and BEM



For the BEM solution, we use a mesh with $h = \lambda/20$





Validation against numerical solution

VTCR against SEA

VTCR	SEA
Calculation of the **response** of one **single** system	Estimation of the **mean value** and **variance** of the **energy** over an **ensemble of systems**
Force or displacement excitation	Power input
Exact solution of governing equation: small **pollution** and **dispersion** error	Requires weak coupling between subsystems and presence of diffuse field within the subsystem

VTCR against SEA

The comparison of SEA and VTCR has two main difficulties

1) Need to consider an ensemble of system
2) Need to normalise solution to obtain constant power input
3) Need to ensure the presence of a diffuse field

The ensemble of plates

Reference plate:

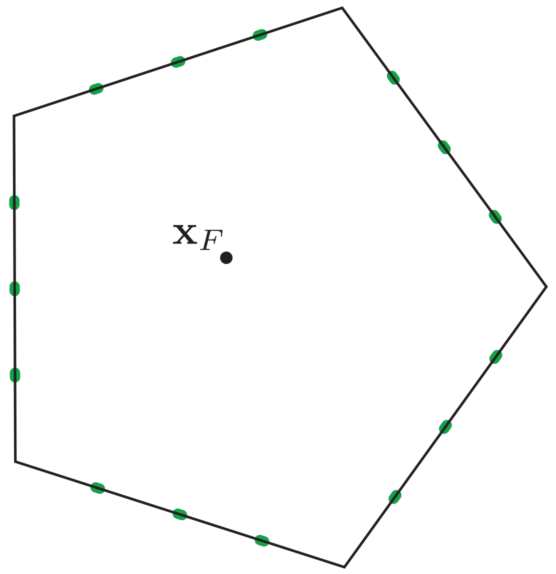
Material:
- Aluminium 1mm thick

Geometry:
- $2m^2$ pentagon

Boundary conditions:
- free edges with 3 equi-distributed pinned points

Loading:
- point force

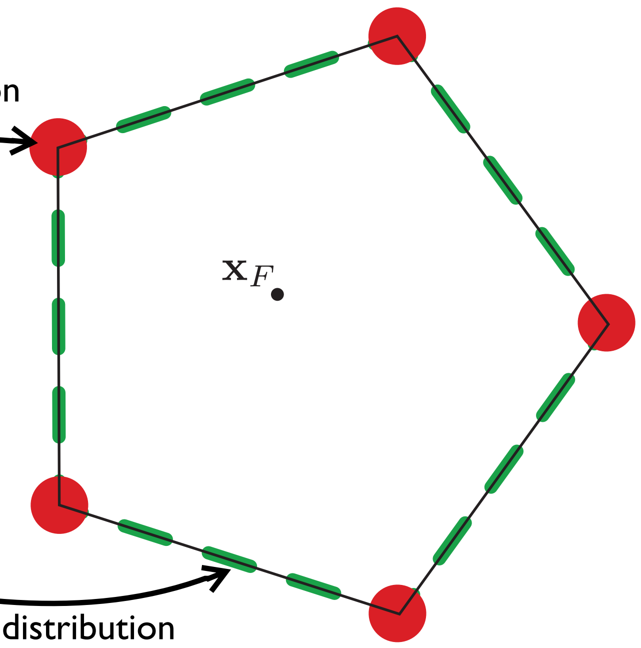
Damping:
- 0.01 to 5%



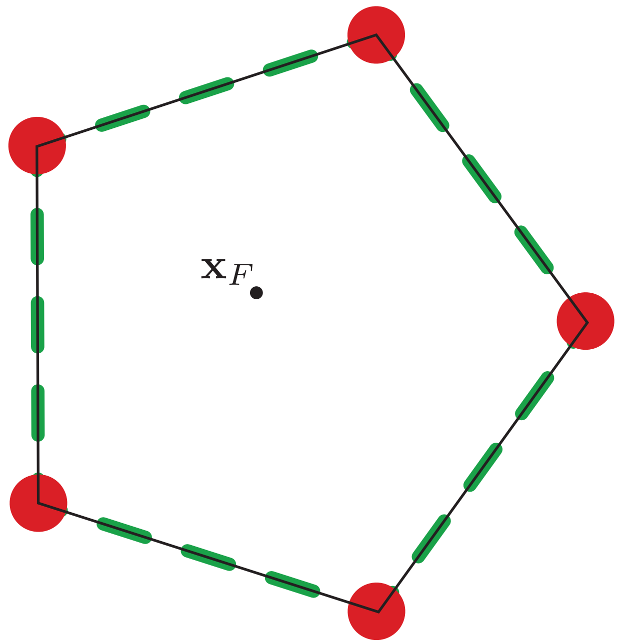
Ensemble of plates:

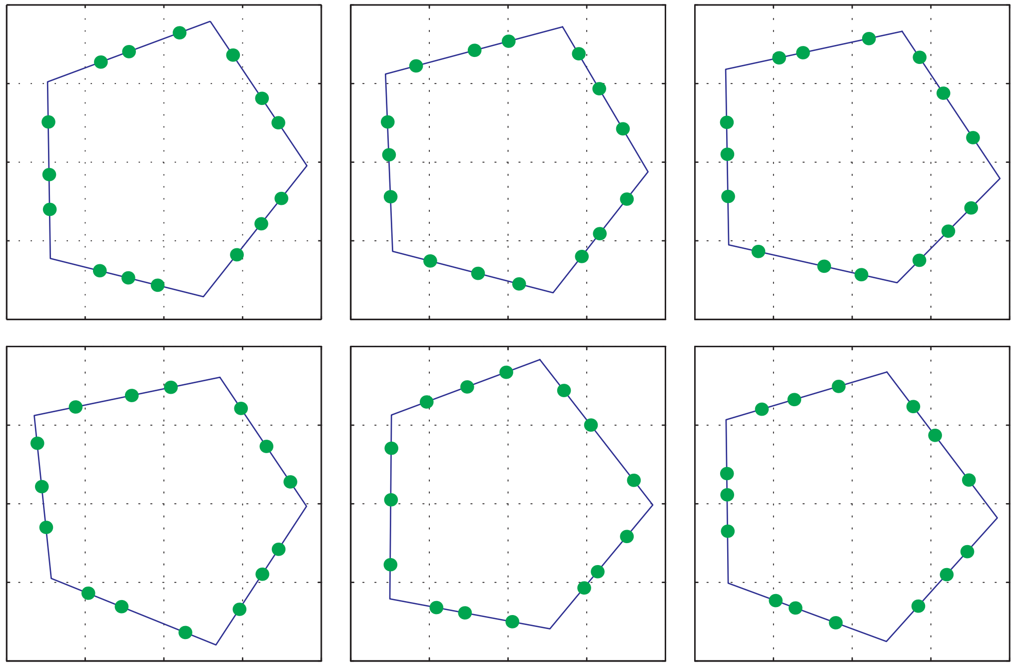
Geometry
- $2m^2$ non-regular pentagon

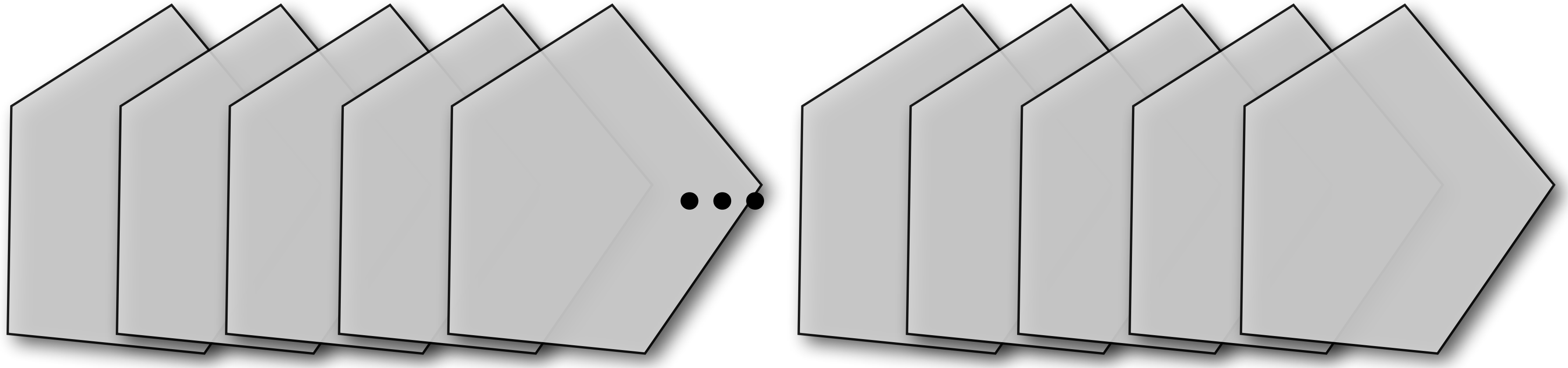
Boundary condition:
- free edges with 3 random pinned points



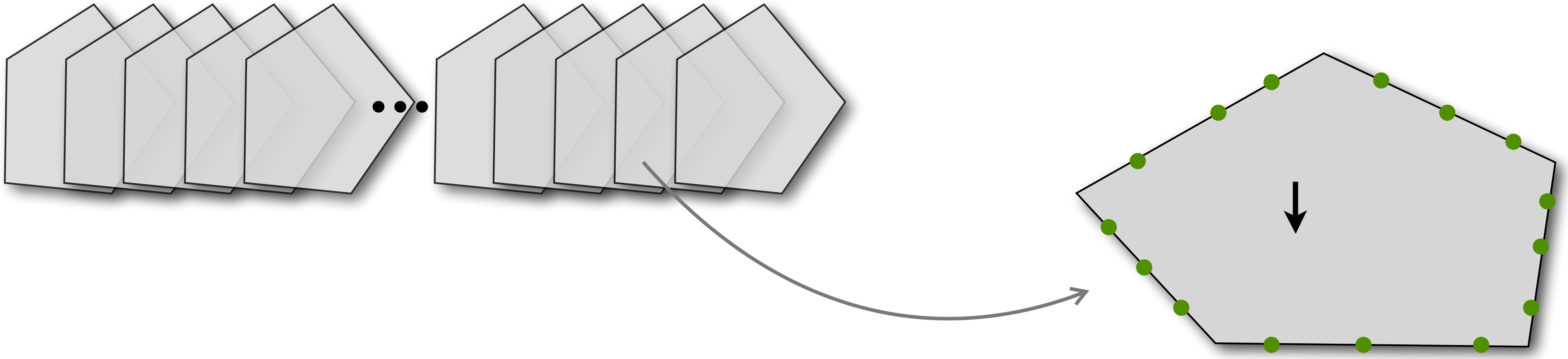
The ensemble of plates

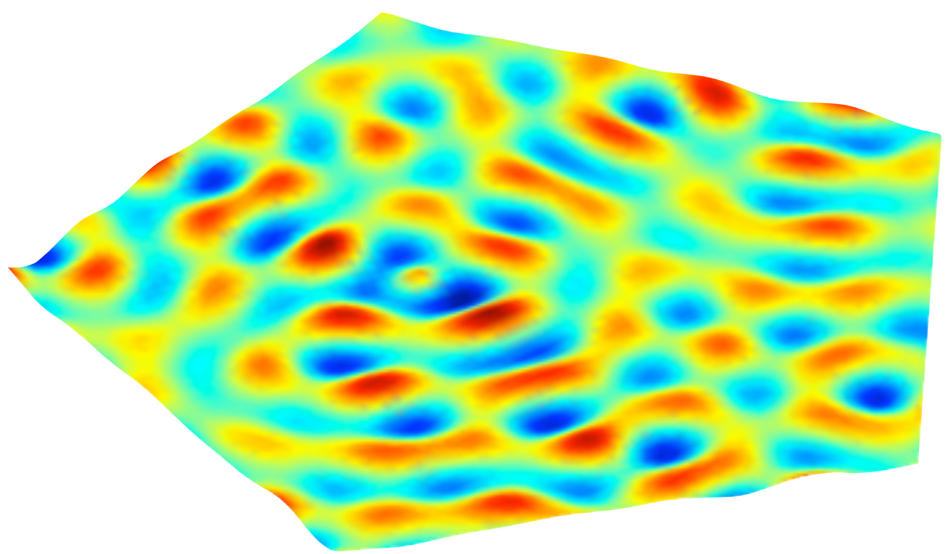




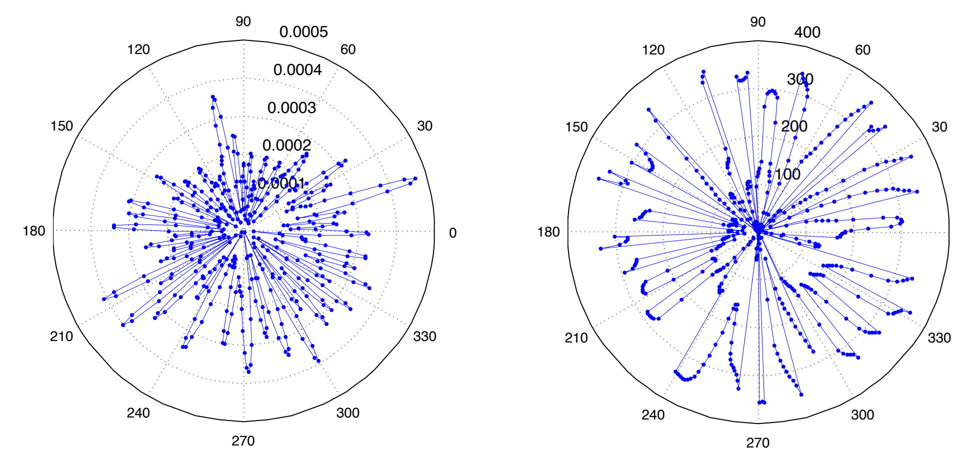


The ensemble of plates: simulation





displacement field



magnitude

phase

waves amplitudes distribution

Definition of a diffuse field

Displacement approach

"a diffuse field is shown to have correlations equal to the Green's function of the body"

O. I. Lobkis and R. L. Weaver, *On the emergence of the Green's function in the correlations of a diffuse field*, JASA 2001

$$\frac{E[w(0) \cdot w(r)^*]}{\sqrt{E[w(0) \cdot w(0)^*]E[w(r) \cdot w(r)^*]}} = J_0(kr)$$

Diffuse field: displacement approach

"a diffuse field is shown to have correlations equal to the Green's function of the body"

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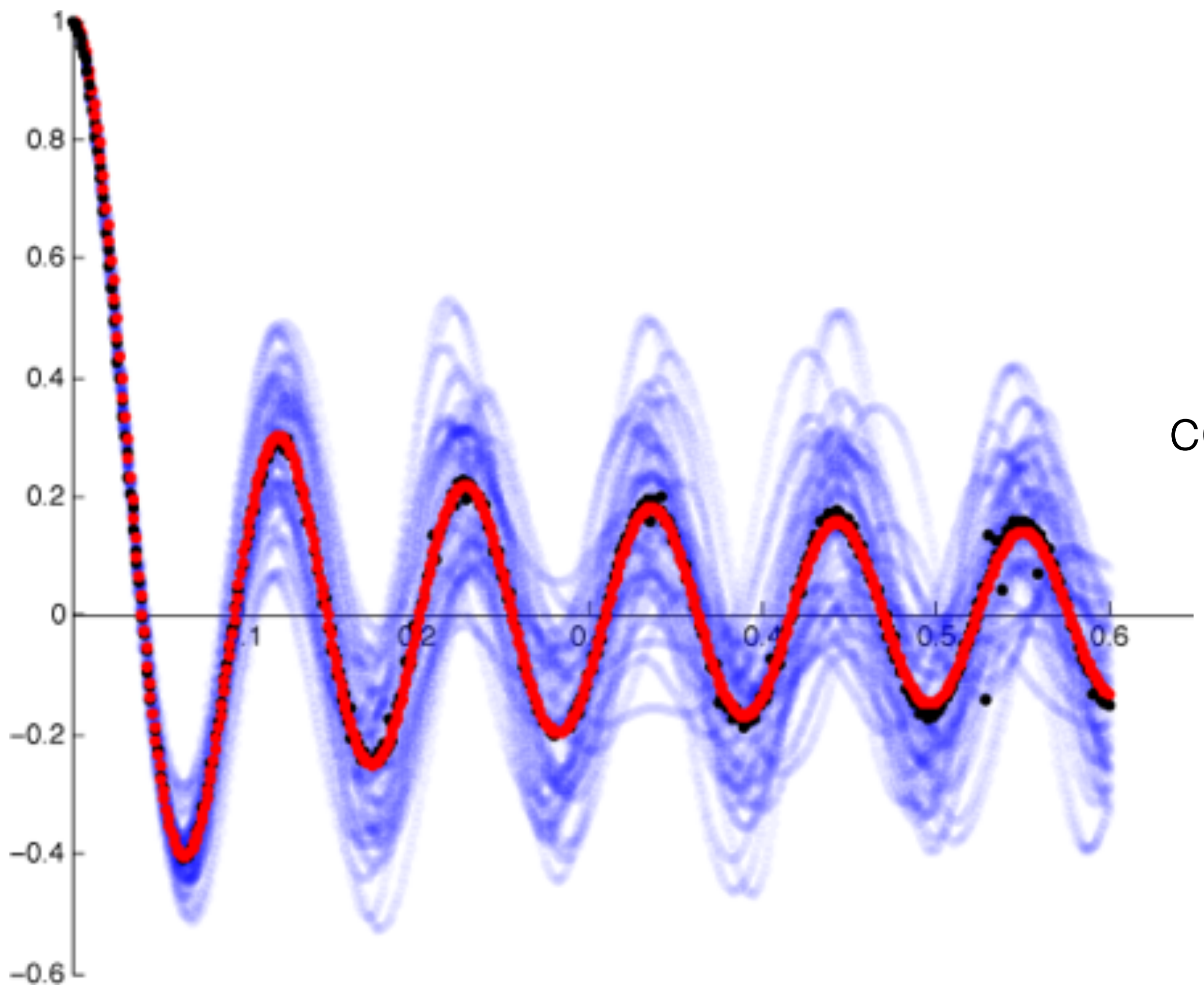
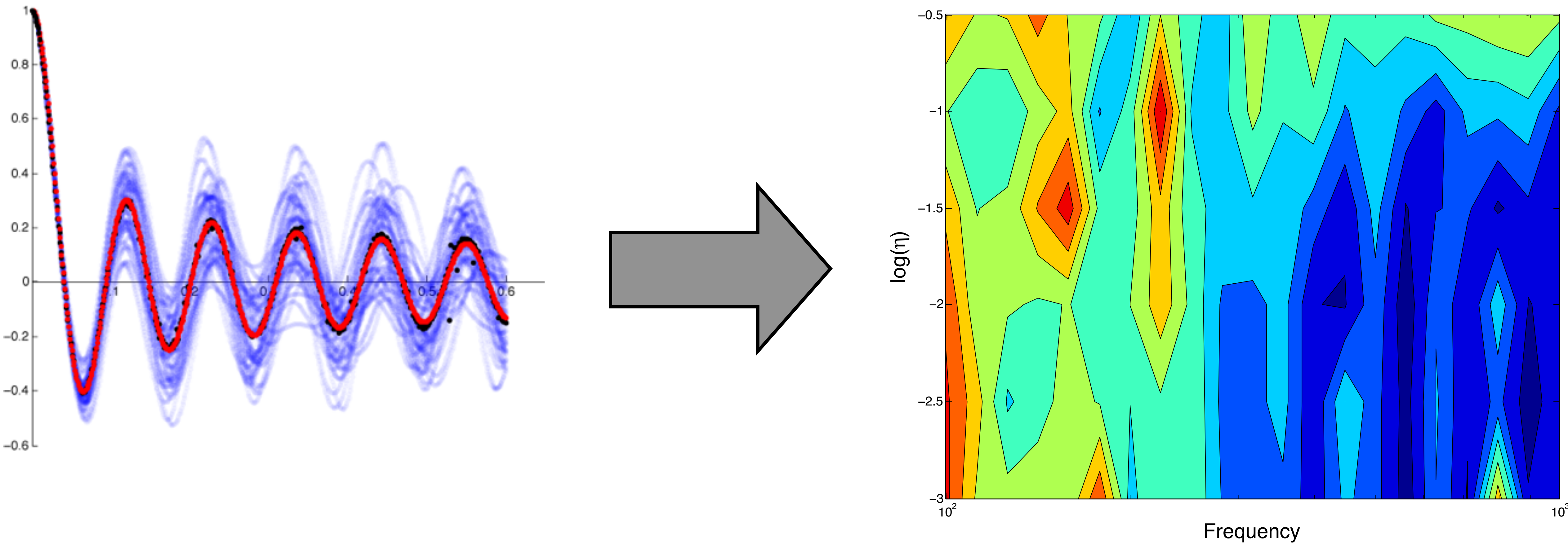
$$\frac{E[w(0) \cdot w(r)^*]}{\sqrt{E[w(0) \cdot w(0)^*]E[w(r) \cdot w(r)^*]}} = J_0(kr)$$



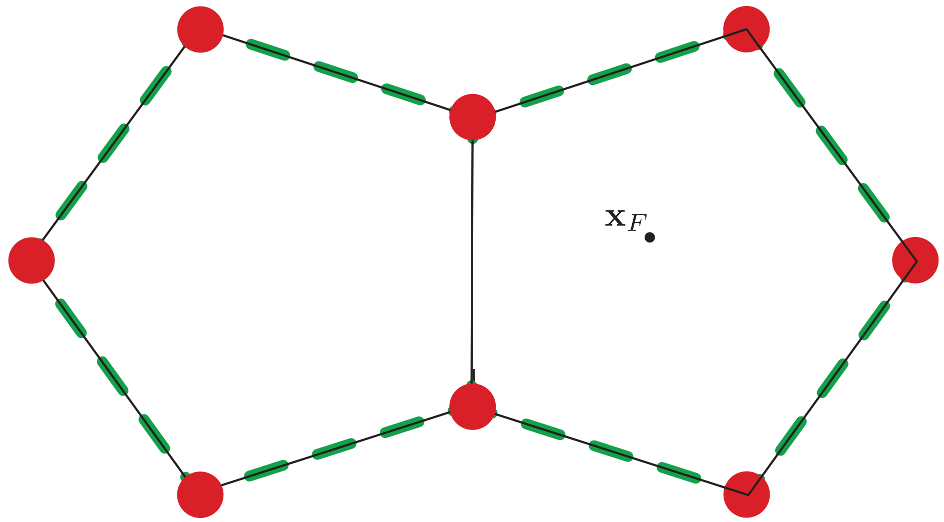
The size of the ensemble is chosen such as the correlation function is close to the Green's Function

Diffusivity of the displacement field across an ensemble of plate.



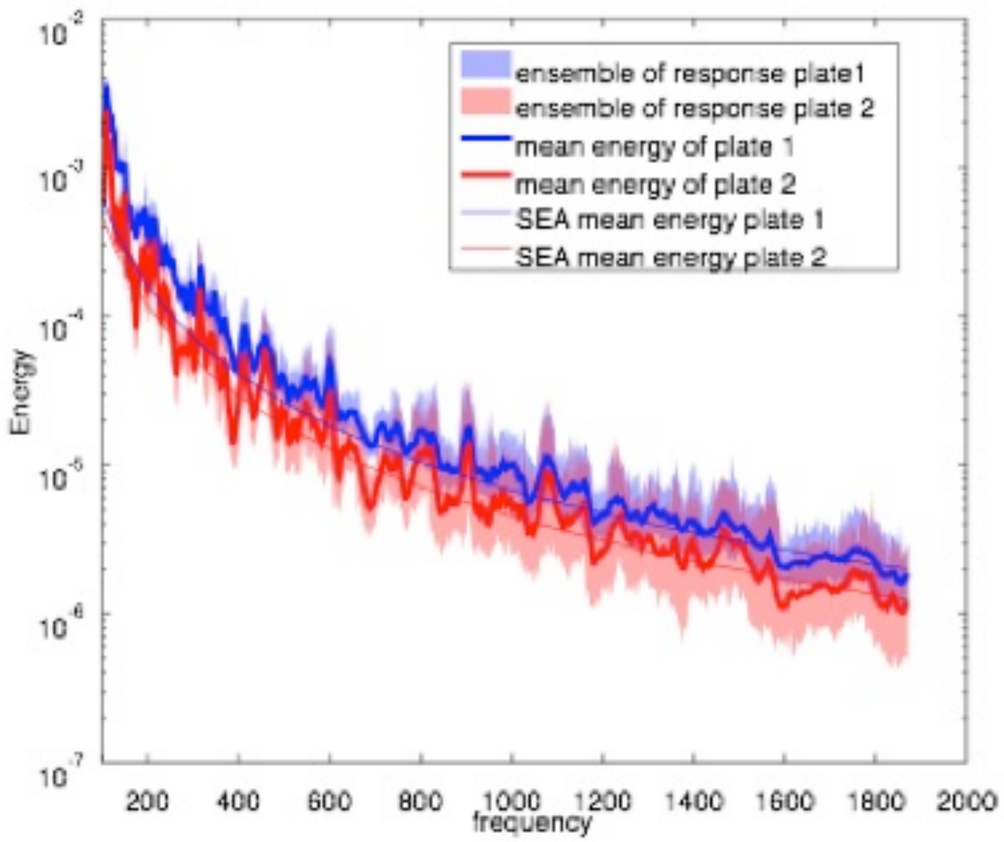
VTCR against SEA

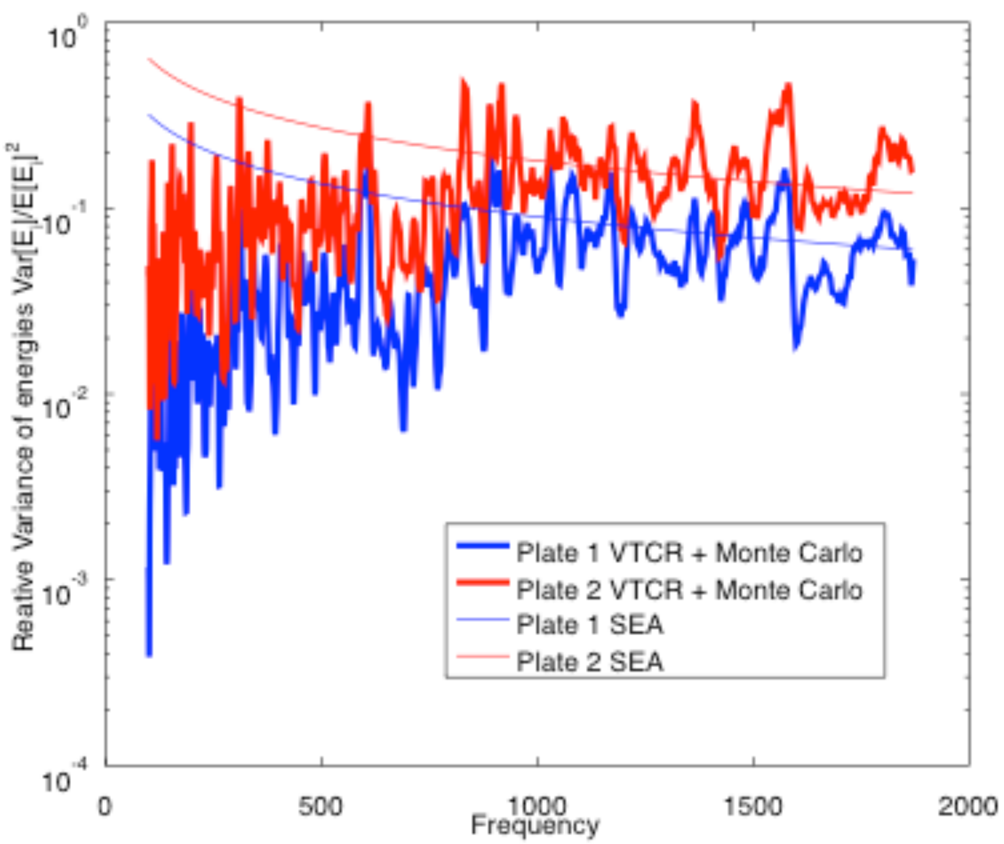
A two coupled plates problem



128 geometries x 64 boundary conditions = 8192 plates

Normalise the deterministic solution such as the power input is 1





How to validate a method?

1) **run some sanity checks**
   a) different model of the same problem
   b) effect of damping
2) **compare to an reference solution**
   a) you have to make sure you are solving the same problem
   b) you have to make sure that the hypothesis of all the two methods are fulfilled

How to define a good benchmark?

a) rigorous definition of the problem…
b) increasing complexity
c) rigorous definition of the quantity of interest
d) requires communication!

Seems obvious, but unfortunately not done in practice…