SmEdA

Statistical modal Energy distribution Analysis

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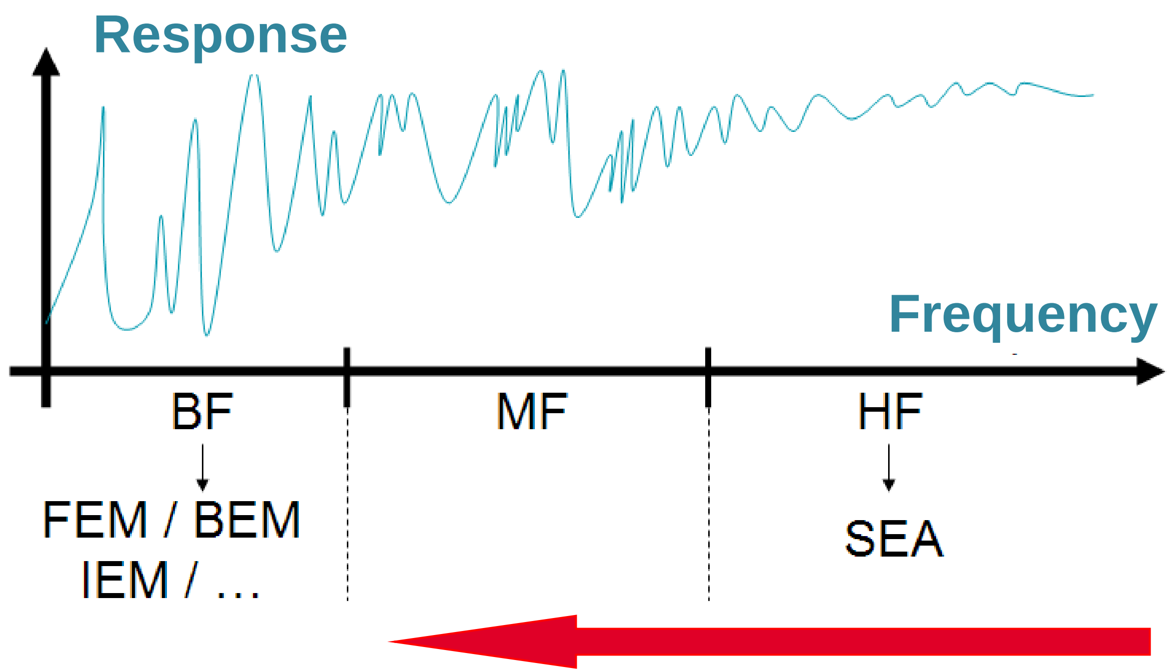
Vibro-acoustic modelling of complex mechanical structures
in the mid-frequency range



SmEdA

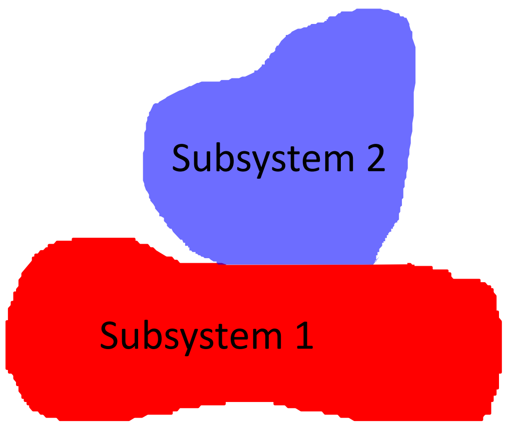
Statistical modal Energy distribution Analysis

~~Up2HF~~ Down2LF summer school!

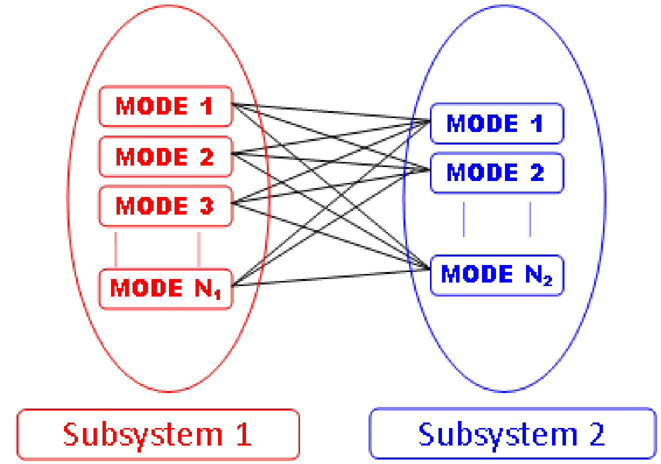
LVA colleagues involved in SmEdA: Jean-Louis Guyader, Nicolas Totaro, Kerem Ege, HaDong Hwang, Youssef Gerges.

L. Maxit – Reformulation and extension of SEA model by taking the modal energy distribution into account, PhD thesis, INSA-Lyon, 2000 (in French!).

Outline of the presentation



Two coupled subsystems excited by white noise forces in $[\Omega_1, \Omega_2]$



Modal coupling schema suggested by SEA

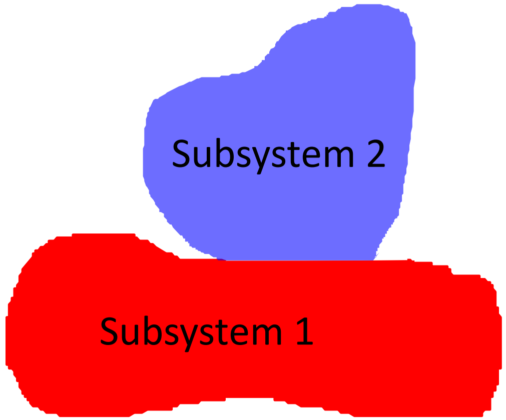
$$\begin{cases} M_p\left[a_p''(t)+\Delta_p a_p'(t)+\omega_p{}^2 a_p(t)\right]-\sum_{r=1}^{N_2} W_{pr} b_r'(t)=F_p(t), & \forall p\in[1,N_1] \\ M_q\left[b_p''(t)+\Delta_p b_p'(t)+\omega_p{}^2 b_p(t)\right]+\sum_{s=1}^{N_1} W_{sq} a_s'(t)=F_q(t), & \forall q\in[1,N_2] \end{cases}$$

Subsystem modes ~ Oscillators

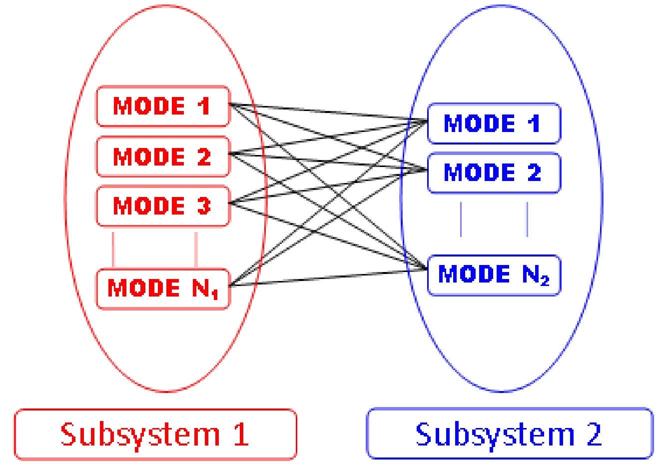
Mode couplings with the others subsystem modes

Resonant modes

I. Dual Modal Formulation (DMF) for coupled subsystems

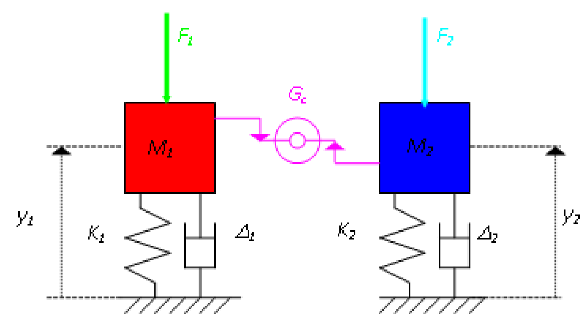


Two coupled subsystems excited by white noise forces in $[\Omega_1, \Omega_2]$



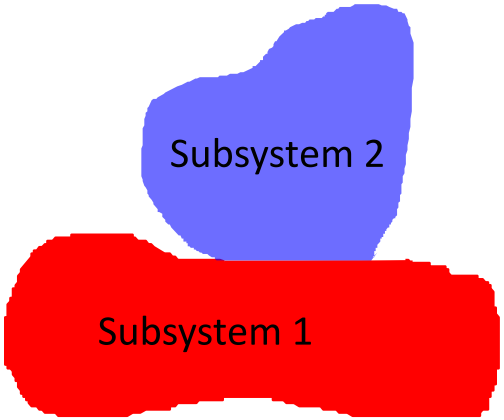
Modal coupling schema suggested by SEA

$$\begin{cases} M_1\left[a_1''(t)+\Delta_1 a_1'(t)+\omega_1{}^2 a_1(t)\right] - G_c b_2'(t) = F_1(t) \\ M_2\left[b_2''(t)+\Delta_2 b_2'(t)+\omega_2{}^2 b_2(t)\right] + G_c a_1'(t) = F_2(t) \end{cases}$$



$$P_{12} = \beta\left(E_1 - E_2\right)$$

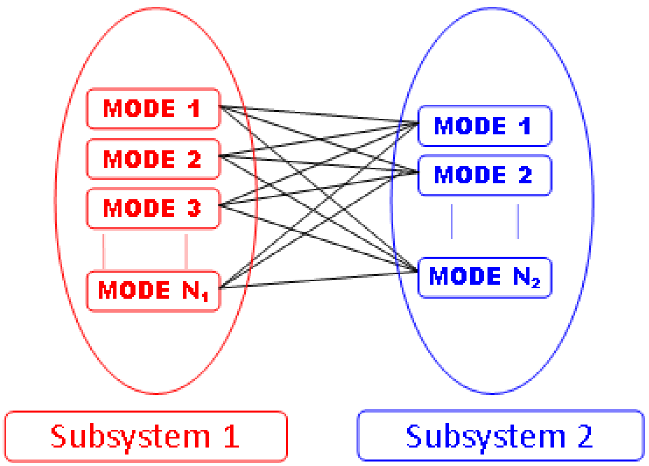
➔ SEA or SmEdA models



Two coupled subsystems excited by white noise forces in $[\Omega_1, \Omega_2]$

Formulation

Cavity-Structure : [Fahy69,70]
Two coupled rods : [Karnopp64]
General case : DMF, [*]

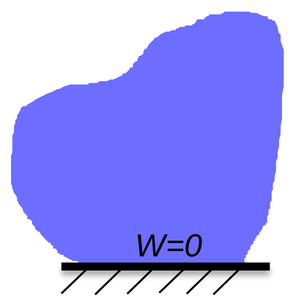


Modal coupling schema suggested by SEA

DMF

Uncoupled – blocked subsystem
➔ Blocked normal modes

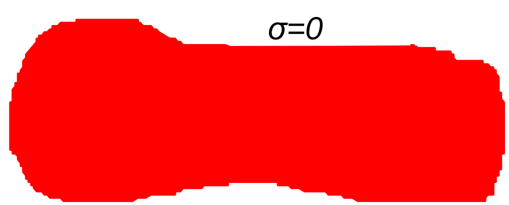
$$\left(\omega_q, \sigma_{ij}^q\right), \forall q \in [\![1, N_2]\!]$$



$$\sigma_{ij}(M_2, t) = \sum_{q=1}^{N_2} b_q(t)\sigma_{ij}^q(M_2)$$

Uncoupled – free subsystem
➔ Free normal modes

$$\left(\omega_p, W_i^p\right), \forall p \in [\![1, N_1]\!]$$

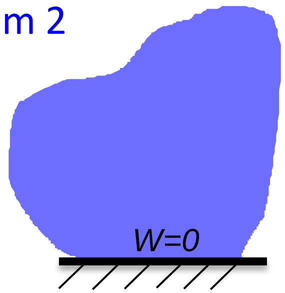


$$W_i(M_1, t) = \sum_{p=1}^{N_1} a_p(t)W_i^p(M_1)$$

[*] L. Maxit, J.L. Guyader - Estimation of sea coupling loss factors using a dual formulation and fem modal information, part 1 : theory. *Journal of Sound and Vibration,* 239 (2001) 907-930.

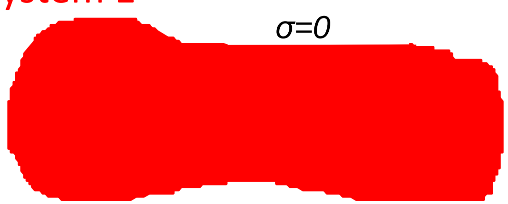
I. Dual Modal Formulation (DMF) for coupled subsystems

Uncoupled – blocked subsystem 2



$$\sigma_{ij}(M_2,t) = \sum_{q=1}^{N_2} b_q(t)\sigma_{ij}^q(M_2)$$

Uncoupled – free subsystem 1



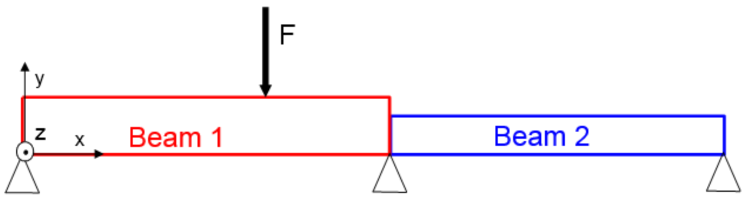
$$W_i(M_1,t) = \sum_{p=1}^{N_1} a_p(t)W_i^p(M_1)$$

$$\begin{cases} M_p\left[a_p''(t)+\Delta_p a_p'(t)+\omega_p{}^2 a_p(t)\right]-\sum_{r=1}^{N_2} W_{pr} b_r'(t) = F_p(t), & \forall p \in [1, N_1] \\ M_q\left[b_p''(t)+\Delta_p b_p'(t)+\omega_p{}^2 b_p(t)\right]+\sum_{s=1}^{N_1} W_{sq} a_s'(t) = F_q(t), & \forall q \in [1, N_2] \end{cases}$$

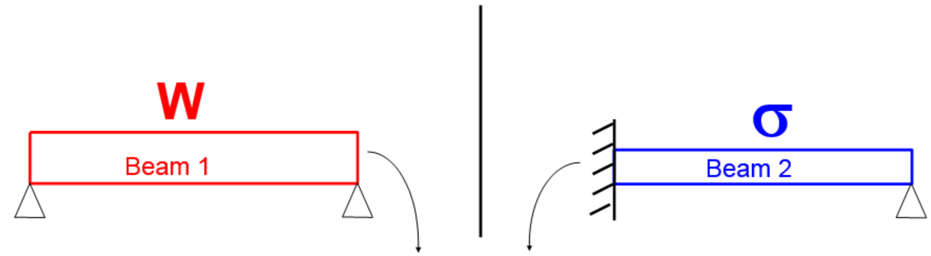
Intermodal works: $W_{pq} = \int_{S_c} W_i^p \sigma_{ij}^q n_j dS$

Coupling surface

First example



Two coupled beams



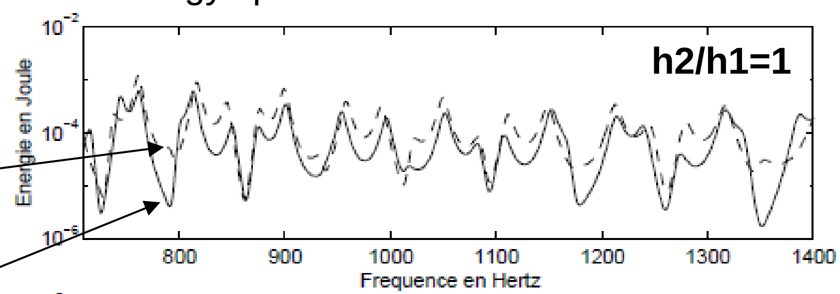
$$\mathbf{W}^{12}_{pq} = \underline{\tilde{\theta}_z^{1p}(L_1)}\,\underline{\tilde{\mathbf{M}}_f^{2q}(L_1)}$$

Uncoupled subsystem modes and intermodal works

Energy spectrum for the non excited beam

Dash, DMF with only the **resonant** modes

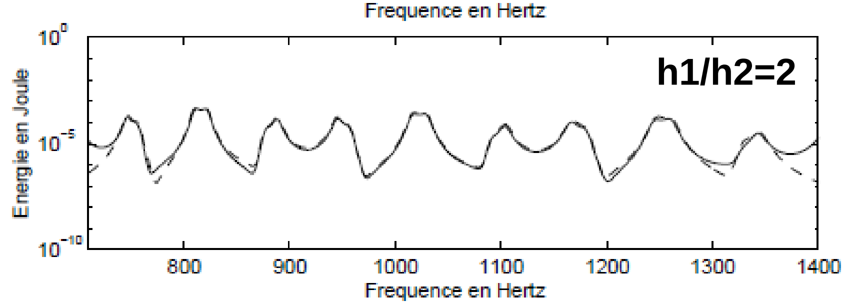
Full, Reference



Energy level for the 1000 Hz octave band

Ref.: 115.8 dB

DMF.: 117.6 dB

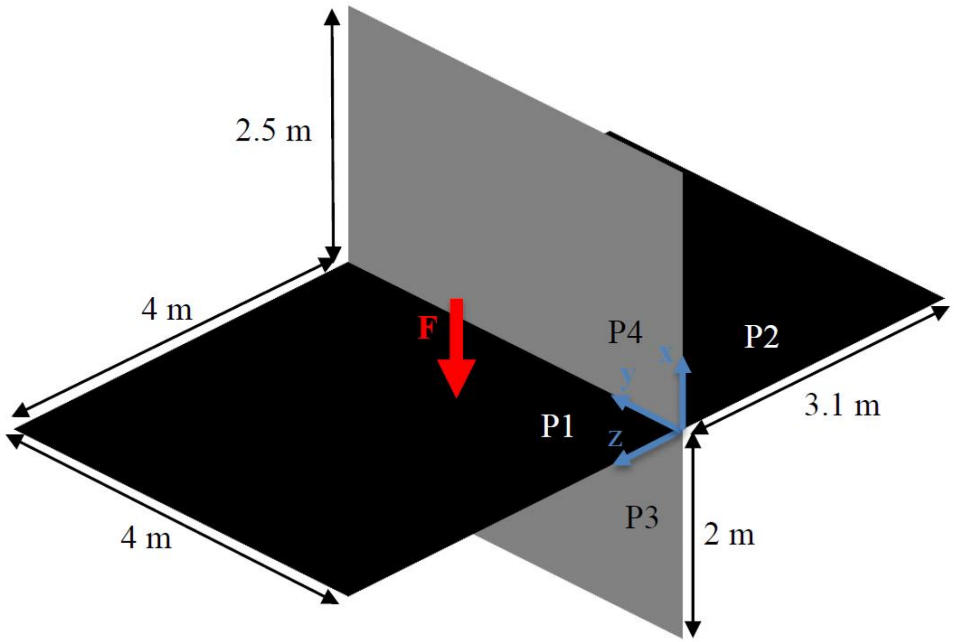


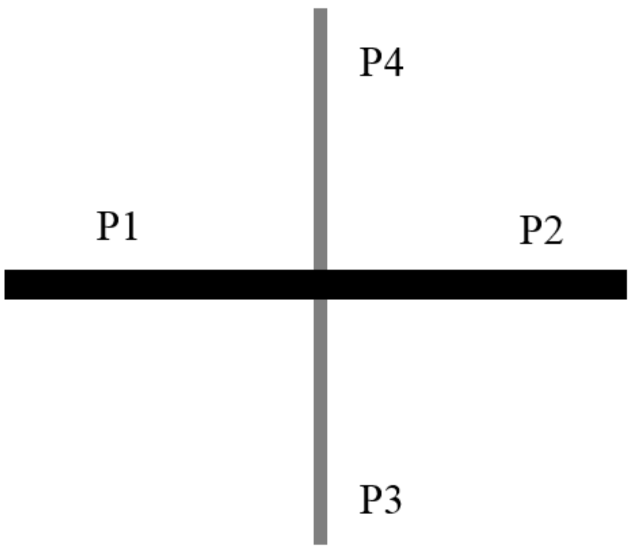
Ref.: 112.7 dB

DMF.: 112.3 dB

➔ a mechanical impedance mismatch is need

Second example





4 panels coupled together at right angle excited by a point force.

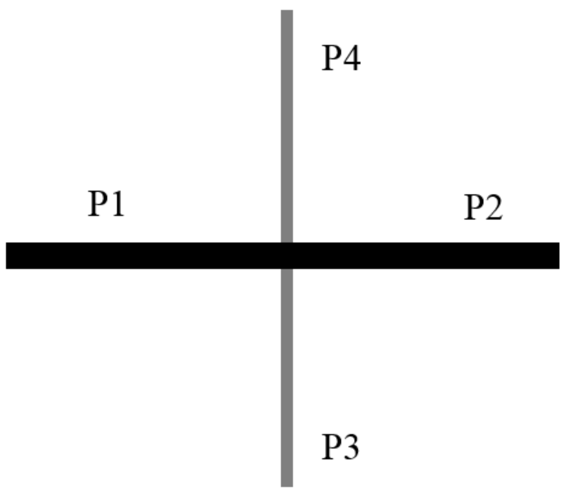
Case	Floor material	Floor thickness	Wall material	Wall thickness
1	Concrete	0.2 m	Brickwork	0.04 m
2	Concrete	0.2 m	Concrete	0.2 m

I. Dual Modal Formulation (DMF) for coupled subsystems

SEA weak coupling assumption (Fahy and James, JSV 190 (1996)):

“*Under the conditions of weak coupling, the system modes are ‘localised’ in the sense that* ***they closely resemble in natural frequency and shape the modes of the uncoupled subsystems*** *(given the appropriate boundary conditions)…*

… Depending on the nature of the coupling, the boundary conditions for the uncoupled system do not always correspond to free displacement at the coupling”.



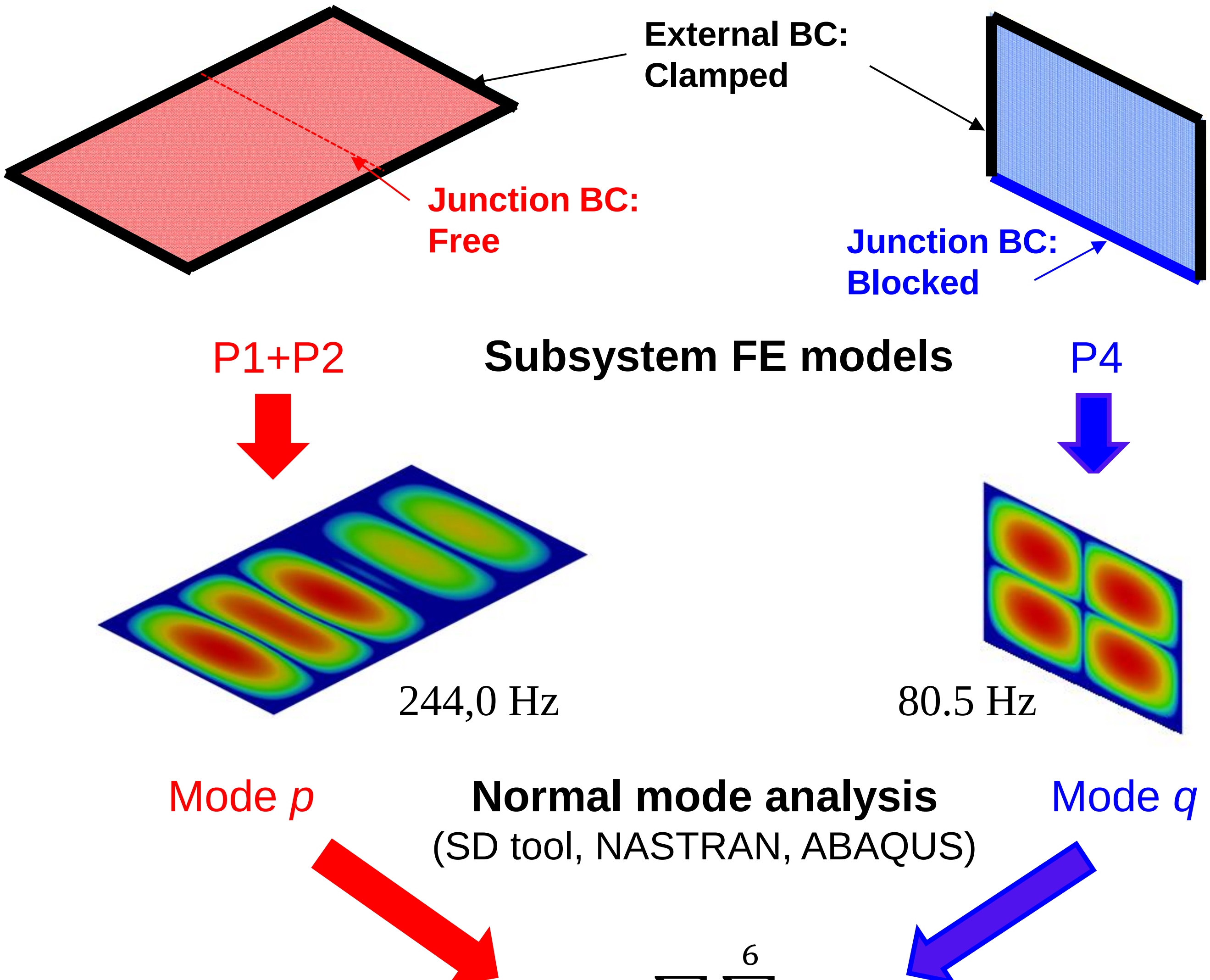
Section of the coupled 4 panels
Test case 1

(a) Free subsystem (b) Blocked subsystem

DMF subsystems

I. Dual Modal Formulation (DMF) for coupled subsystems

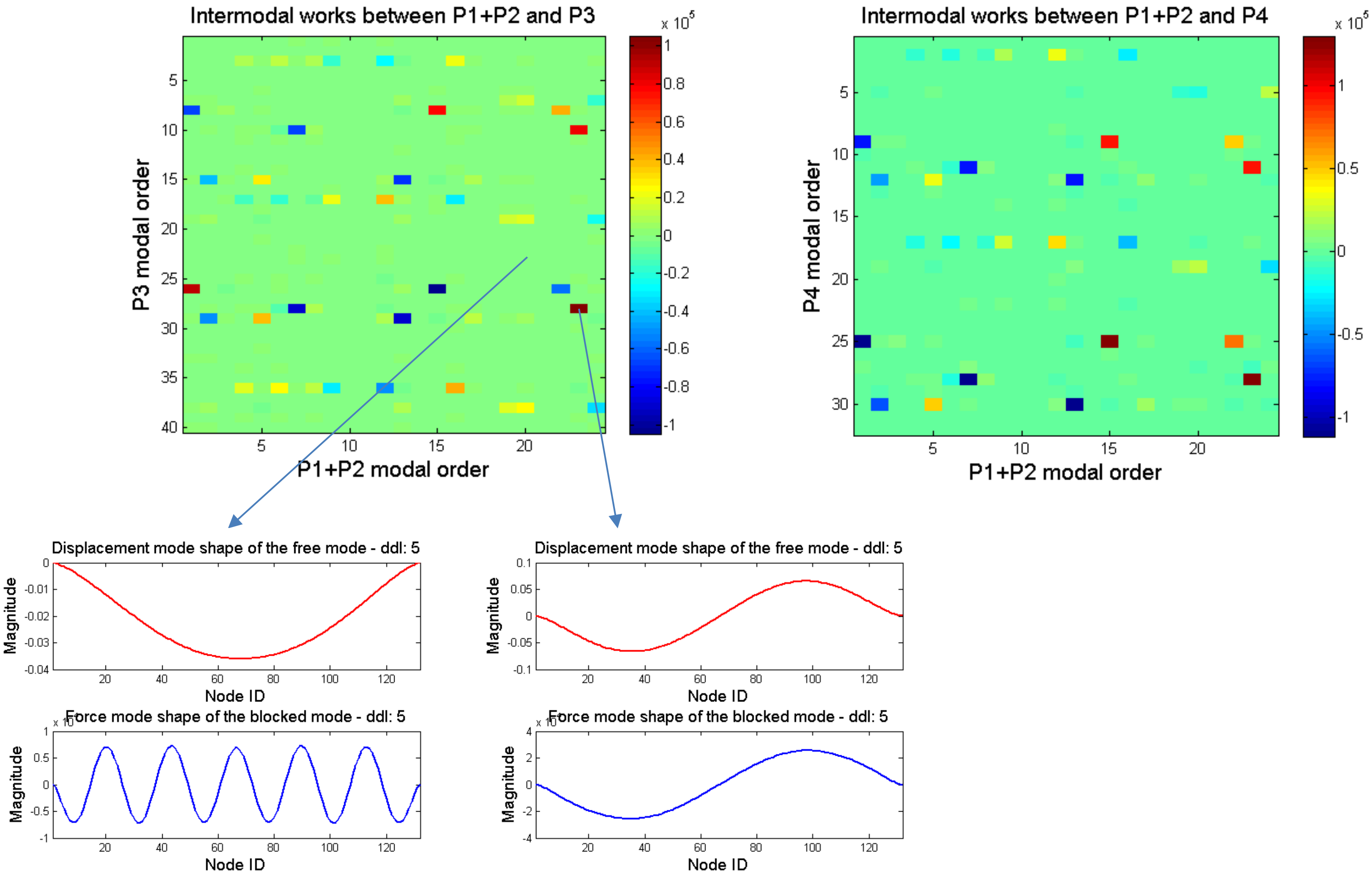
Illustration of the numerical process for intermodal work estimation:



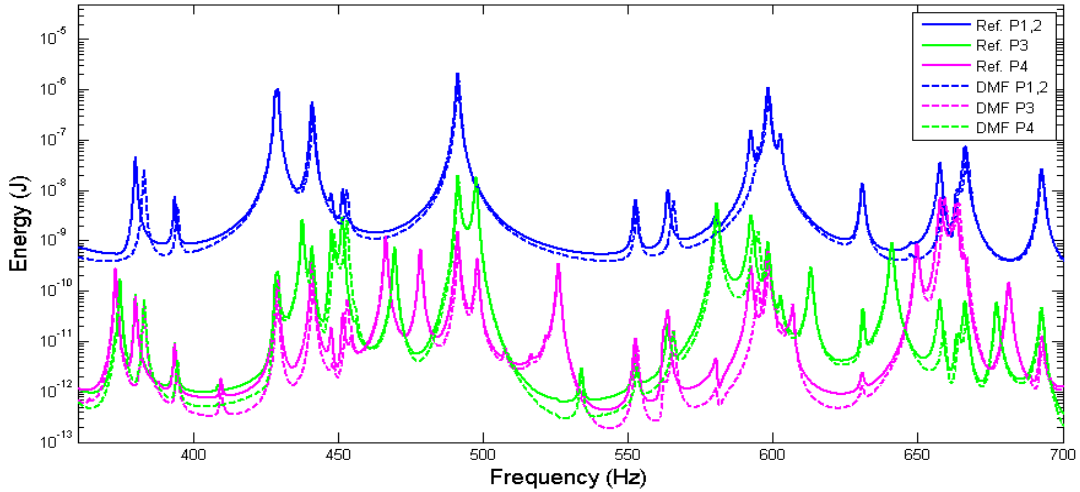
$$W_{pq} = \sum_{i \in I} \sum_{k=1}^{6} u_{p,k}^{i} \, f_{q,k}^{i}$$

I. Dual Modal Formulation (DMF) for coupled subsystems

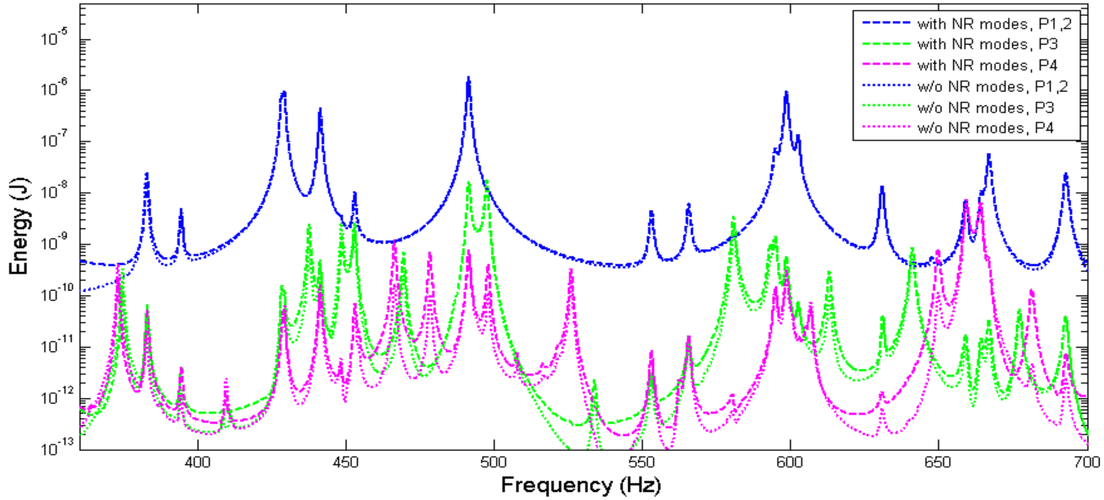
Intermodal works for resonant modes in the 500 Hz Octave band



Subsystem energy response in function of the frequency. Test case n°1.



Comparison between reference (full) and DMF with Non-Resonant (NR) modes (dash)

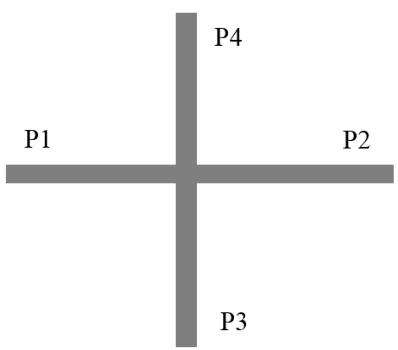


Comparison between DMF with NR modes (dash) and DMF without NR modes (dotted)

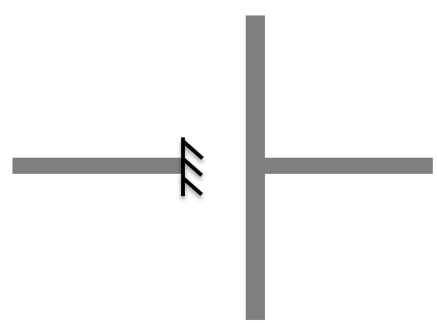
I. Dual Modal Formulation (DMF) for coupled subsystems

	Panel \ Energy	1+2	3	4
Test case 1	Reference	79.0 dB (78.2+71.6)	58.9 dB	55.0 dB
	DMF	78.3 dB	58.2 dB	54.8 dB
	DMF (resonant modes only)	78.3 dB	57.9 dB	54.2 dB
Test case 2	Reference	78.6 dB (77.8+70.9)	67.2 dB	68.0 dB
	DMF	78.8 dB	45.2 dB	45.6 dB
	DMF (resonant modes only)	78.8 dB	42.9 dB	42.5 dB

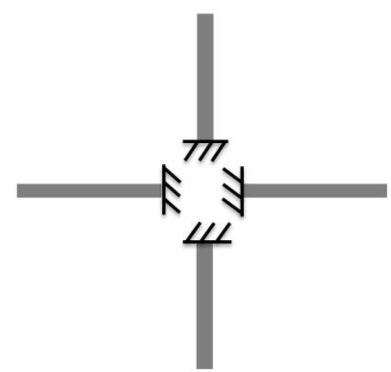
Comparison of subsystem energies obtained with different calculations.
Results for the octave band 500 Hz (dB, ref. 10^{-12}J).



Section of the coupled 4 panels
Test case 2



Alternative DMF
substructuring



Craig –Bampton
substructuring … under study

L. Maxit, Investigation for obtaining a modal coupling scheme in agreement with the SmEdA/SEA models for multi-subsystems connected at a junction, *Proceedings of NOVEM 2015*, Dubrovnik, Croatia, April 2015. 22 p.

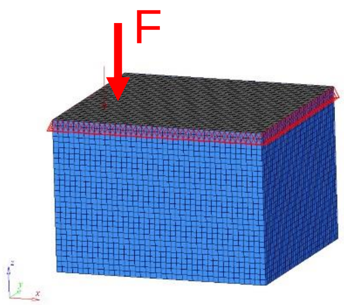
I. Dual Modal Formulation (DMF) for coupled subsystems

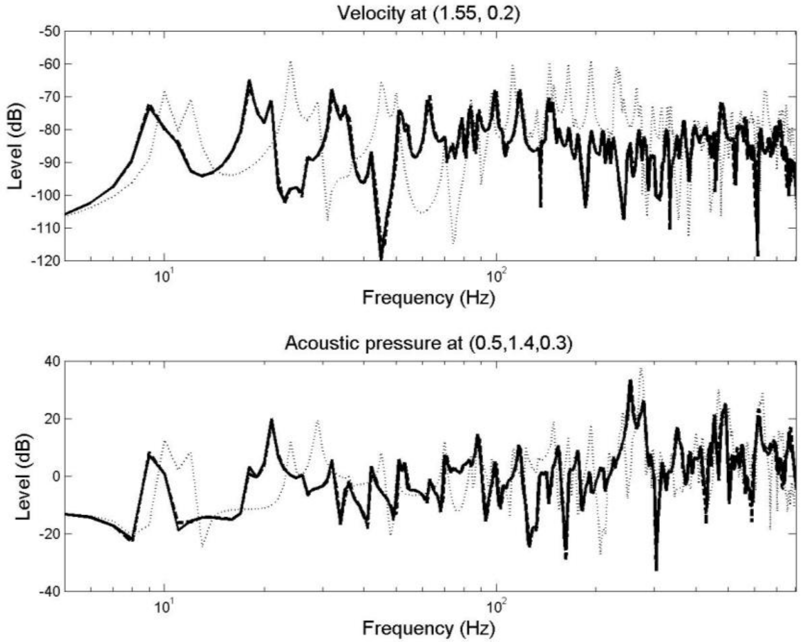
Third example: a rectangular plate coupled to a cavity

- Case of the cavity filled of air

  ➔ Good description with DMF taking only the resonant modes

- Case of the cavity filled of water (heavy fluid)

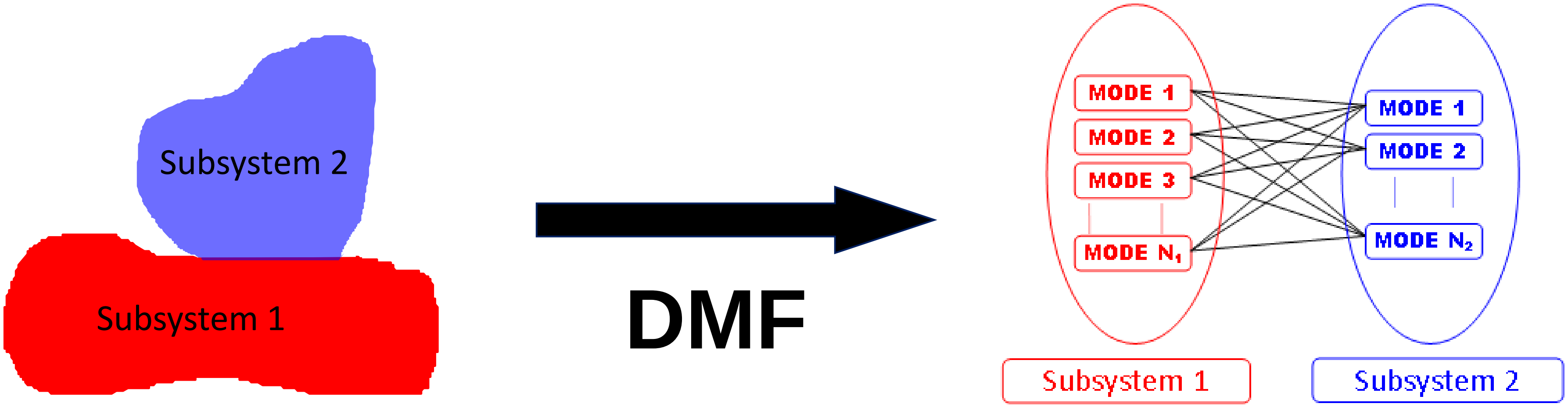




Comparison between reference (full), DMF without Non-Resonant modes (dotted), and DMF with Non-Resonant (NR) modes (dash)

L. Maxit, Analysis of the modal energy distribution of an excited vibrating panel coupled with a heavy fluid cavity by a dual modal formulation. *Journal of Sound and Vibration*, vol. 332, p. 6703-6724, 2013.

I. Dual Modal Formulation (DMF) for coupled subsystems



Two coupled subsystems excited by broad band excitations

Interaction between two sets of resonant subsystem modes

A mechanical **impedance mismatch** between the two subsystems is need.

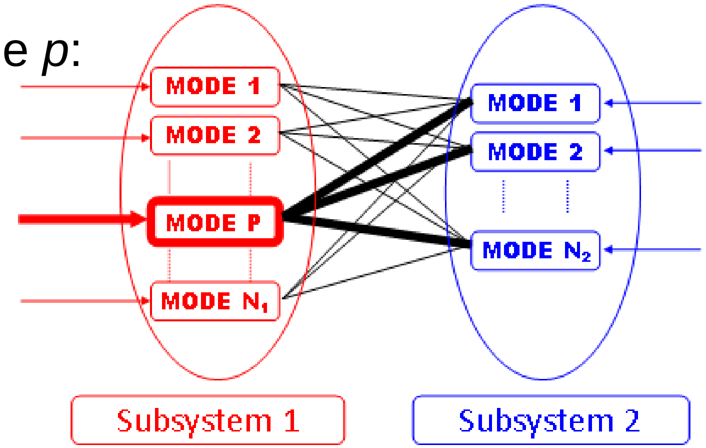
In this case, the **soft** subsystem is represented by its uncoupled-**blocked** modes whereas the other one (i.e. the **stiffer** one) is represented by its uncoupled-**free** modes.

In the following, we suppose to be in these conditions.

SmEdA is based on:

- the modal description of uncoupled subsystems (natural frequencies, mode shapes)
- the same assumptions as SEA except for the modal energy equipartition
- the description of the energy sharing between modes rather than between subsystems

- Power balance mode *p*:



$$\Pi_{inj}^{1p} = \Pi_{diss}^{1p} + \sum_{q'=1}^{N_2} \Pi_{pq'}^{12}$$

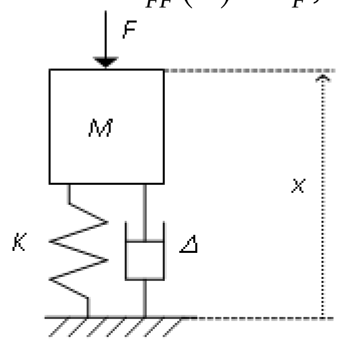
Modal injected power

Modal dissipated power

Powers exchanged with modes of other coupled subsystems

- Time-averaged power for a single oscillator

$$S_{FF}(\omega) = \bar{S}_F, \ \forall \omega \in [0, \infty[$$



An oscillator excited by a white noise force

Equation of motions:

$$M\left[\ddot{x}(t) + \omega_0\eta_0\dot{x}(t) + (\omega_0)^2 x(t)\right] = F(t)$$

Time-averaged energy (kinetic + strain):

$$< E_t >_t = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{+T} \left\{ \frac{1}{2} M \left[\dot{x}(t) \right]^2 + \frac{1}{2} K \left[x(t) \right]^2 \right\} dt$$

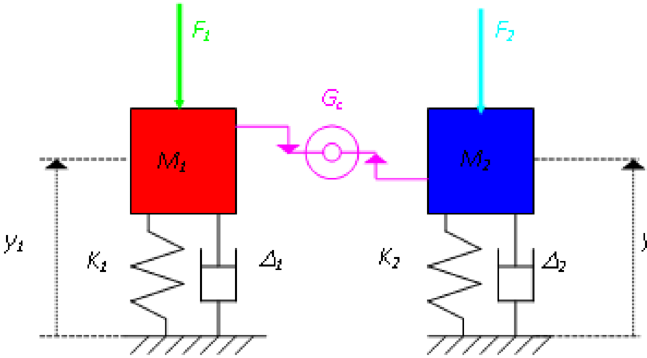
$\sim$ (when time-averaged)

$$< P_{diss} >_t = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{+T} \omega_0\eta_0 [\dot{x}(t)]^2 \, dt \quad \Rightarrow \quad < P_{diss} >_t \approx \omega_0\eta_0 < E_t >_t \quad (1)$$

$$< P_{inj} >_t = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{+T} F(t)\dot{x}(t) \, dt \quad \Rightarrow \quad < P_{inj} >_t = \frac{\pi \bar{S}_F}{4M} \quad (2)$$

R.H. Lyon, R.G. DeJong *Theory and Application of Statistical Energy Analysis,* Butterworth-Heineman (1995)

- Time-averaged power exchanged by two oscillators:
  Case of white noise forces in [0,∞[



Two oscillators coupled by a gyrostatic element and excited by uncorrelated white noise forces

Equation of motions

$$\begin{cases} \ddot{x}_1(t)+\Delta_1\dot{x}_1(t)+\omega_1^2 x_1(t)-\sqrt{M_1^{-1}M_2}\,\gamma\,\dot{x}_2(t)=F_1(t), \\ \ddot{x}_2(t)+\Delta_2\dot{x}_2(t)+\omega_2^2 x_2(t)+\sqrt{M_1M_2^{-1}}\,\gamma\,\dot{x}_1(t)=F_2(t), \end{cases}$$

$$\gamma = G_c\left(\sqrt{M_1M_2}\right)^{-1}$$

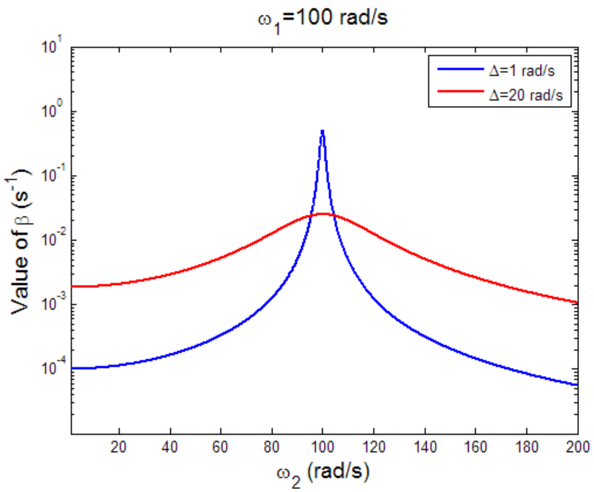
Time-averaged power flow between the two oscillators:

$$< P_{12} >_t = \beta\left(< E_1 >_t - < E_2 >_t\right) \quad (3)$$

with the coupling factor:

$$\beta = \frac{\gamma^2\left(\Delta_1\omega_2^2+\Delta_2\omega_1^2\right)}{\left(\omega_1^2-\omega_2^2\right)^2+\left(\Delta_1+\Delta_2\right)\left(\Delta_1\omega_2^2+\Delta_2\omega_1^2\right)} \quad (4)$$

➔

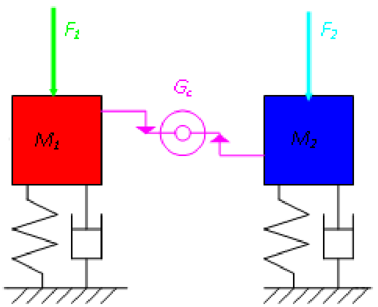


T.D. Scharton, R.H. Lyon, Power flow and energy sharing in random vibration. *Journal of the Acoustical Society of America,* 43, 1332-1343, 1968.

- Time-averaged power exchanged by two oscillators
  Case of white noise forces in $[\Omega_1, \Omega_2]$

Oscillators excited by white noise force in the frequency band [0,∞[

$$\omega_1 = \sqrt{\frac{K_1}{M_1}}$$



$$\omega_2 = \sqrt{\frac{K_2}{M_2}}$$

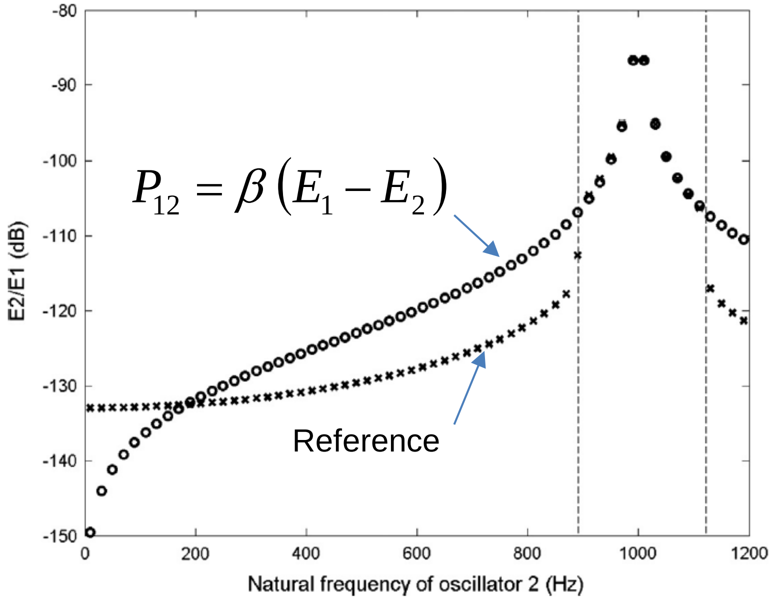
$$P_{12} = \beta\left(E_1 - E_2\right)$$

Oscillators excited by white noise force in the frequency band $[\Omega_1, \Omega_2]$

$$P_{12} \approx \beta\left(E_1 - E_2\right)$$

$$\text{if } \begin{cases} \omega_1 \in [\Omega_1, \Omega_2] \\ \omega_2 \in [\Omega_1, \Omega_2] \end{cases}$$

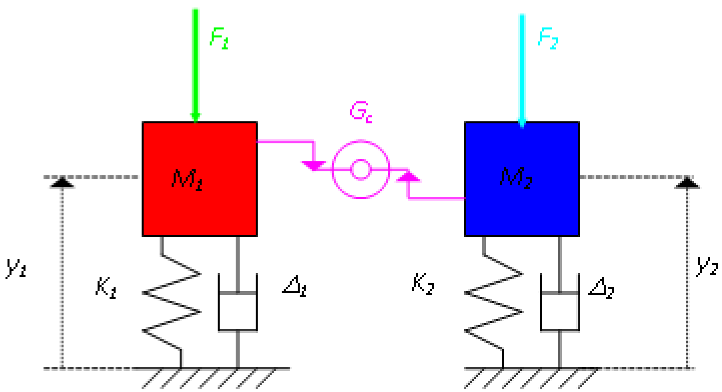
Case of oscillator 1 excited by a white noise force in the 1000 Hz third octave band



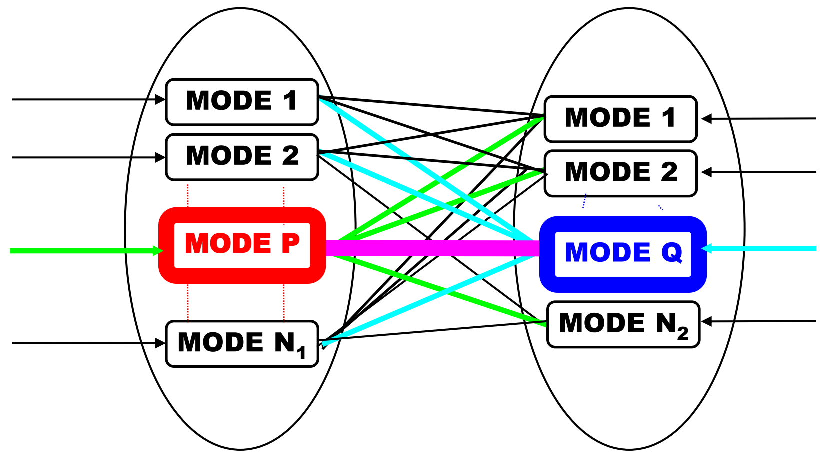
Oscillator energy ratio E2/E1 versus the natural frequency of oscillator 2. Natural frequency of oscillator 1 = 1000 Hz.

Expression adapted for **resonant** modes (only)

- Energy sharing between mode *p* and mode *q*



$$P_{12} = \beta\left(E_1 - E_2\right)$$



$$\Pi_{pq}^{12} = \beta_{pq}^{12}\left(E_p^1 - E_q^2\right)$$

Intermodal Coupling Factors:

Intermodal work

$$\beta_{pq}^{12} = \frac{\left(\mathrm{W}_{pq}^{12}\right)^2}{M_p^1\left(\omega_q^2\right)^2 M_q^2}\left\{\frac{\Delta_p^1\left(\omega_q^2\right)^2 + \Delta_q^2\left(\omega_p^1\right)^2}{\left[\left(\omega_p^1\right)^2 - \left(\omega_q^2\right)^2\right]^2 + \left(\Delta_p^1 + \Delta_q^2\right)\left[\Delta_p^1\left(\omega_q^2\right)^2 + \Delta_q^2\left(\omega_p^1\right)^2\right]}\right\}.$$

Modal frequency　　Modal damping bandwidth

- Modal energy equations of motions (for the two coupled subsystems):

$$\begin{cases} \Pi_{inj}^{1p} = \left(\omega_p^1 \eta_p^1 + \sum_{q'=1}^{N_2} \beta_{pq'}^{12} \right) E_p^1 - \sum_{q'=1}^{N_2} \beta_{pq'}^{12} E_{q'}^2, & \forall p \in [1,\ldots,N_1], \\ \Pi_{inj}^{2q} = - \sum_{p'=1}^{N_1} \beta_{p'q}^{12} E_{p'}^1 + \left(\omega_q^2 \eta_q^2 + \sum_{p'=1}^{N_1} \beta_{p'q}^{12} \right) E_q^2, & \forall q \in [1,\ldots,N_2]. \end{cases}$$

➔ N_1+N_2 equations, N_1+N_2 unknowns ➔ SmEdA

- Subsystem energies:

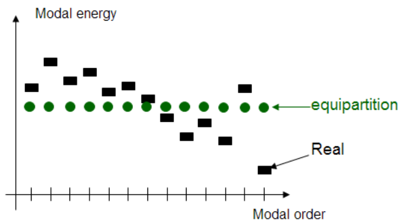
$$E_1 = \sum_{p=1}^{N_1} E_p^1, \quad E_2 = \sum_{q=1}^{N_2} E_q^2$$

- Response in term of physical quantities:

$$< V_\alpha{}^2 >= \frac{E_\alpha}{M_\alpha} \qquad < p_\alpha{}^2 >= \frac{\rho_0 c_0{}^2 E_\alpha}{V}$$

- Relation with SEA-like (Statistical Energy Analysis)

Modal energy equipartition assumption:



$$E_p^1 = e_1, \forall p \in [1,\ldots,N_1], \; E_q^2 = e_1, \; \forall q \in [1,\ldots,N_2]$$

➔ SEA equations (by summing the SmEdA equations):

$$\begin{cases} \Pi_{inj}^1 = \omega_c \eta_1 E_1 + \omega_c \left(\eta_{12} E_1 - \eta_{21} E_2 \right), \\ \Pi_{inj}^2 = \omega_c \eta_2 E_2 + \omega_c \left(\eta_{21} E_2 - \eta_{12} E_1 \right), \end{cases}$$

with the subsystem injected powers,

$$\Pi_{inj}^1 = \sum_{p=1}^{N_1} \Pi_{inj}^{1p}, \; \Pi_{inj}^2 = \sum_{q=1}^{N_2} \Pi_{inj}^{2q},$$ and,

the SEA-like coupling loss factor (CLF),

$$\eta_{12} = \frac{\sum_{p=1}^{N_1} \sum_{q=1}^{N_2} \beta_{pq}^{12}}{N_1 \omega_c}$$

depanding only on the modal information of each uncoupled subsystem.

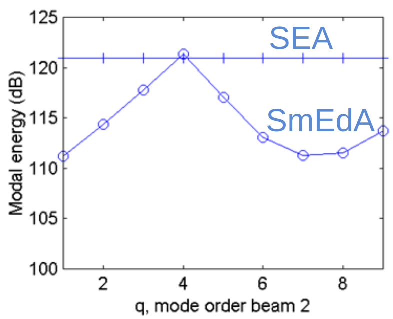
(1) Subsystems with low modal overlap

- ➔Low damping
- ➔Low modal density
- ➔Mid-frequency domain

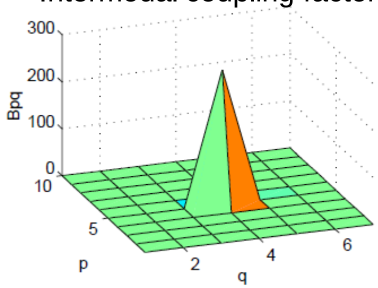
Example on a case studied in the literature : Two coupled beams with varying damping

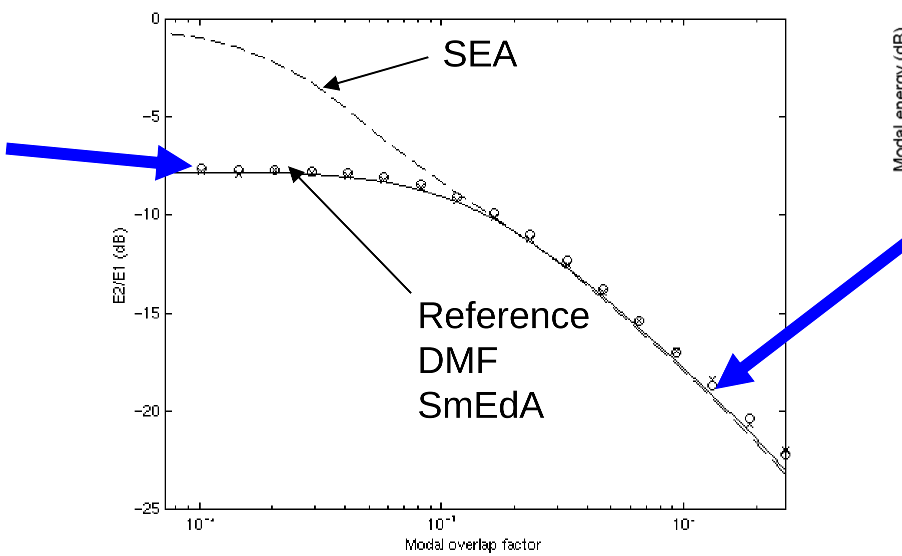
FF. YAP, J. WOODHOUSE - Investigation of damping effects on SEA of coupled structures, JSV, 197 (1996)

Modal energy distribution – beam 2



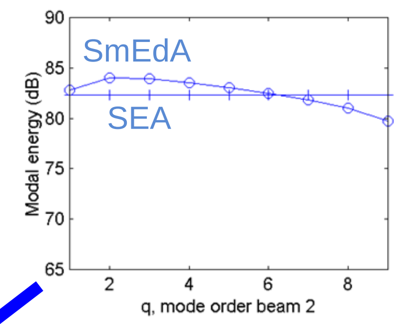
Intermodal coupling factors



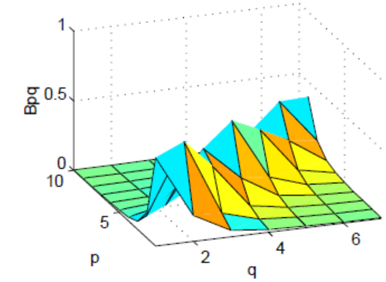


Beam energy ratio versus modal overlap

Modal energy distribution – beam 2



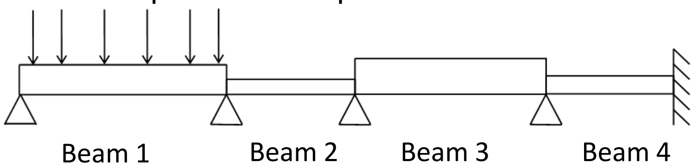
Intermodal coupling factors



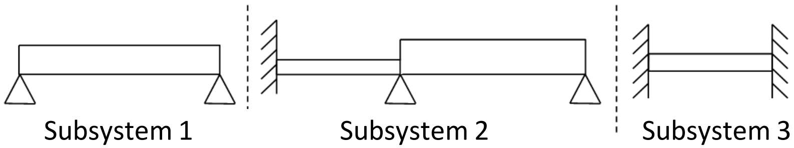
L. Maxit, J.L. Guyader - Extension of SEA model to subsystems with non uniform modal energy distribution. *Journal of Sound and Vibration,* 265 (2003) 337-358.

(2) Heterogeneous subsystems

Example: Four coupled beams

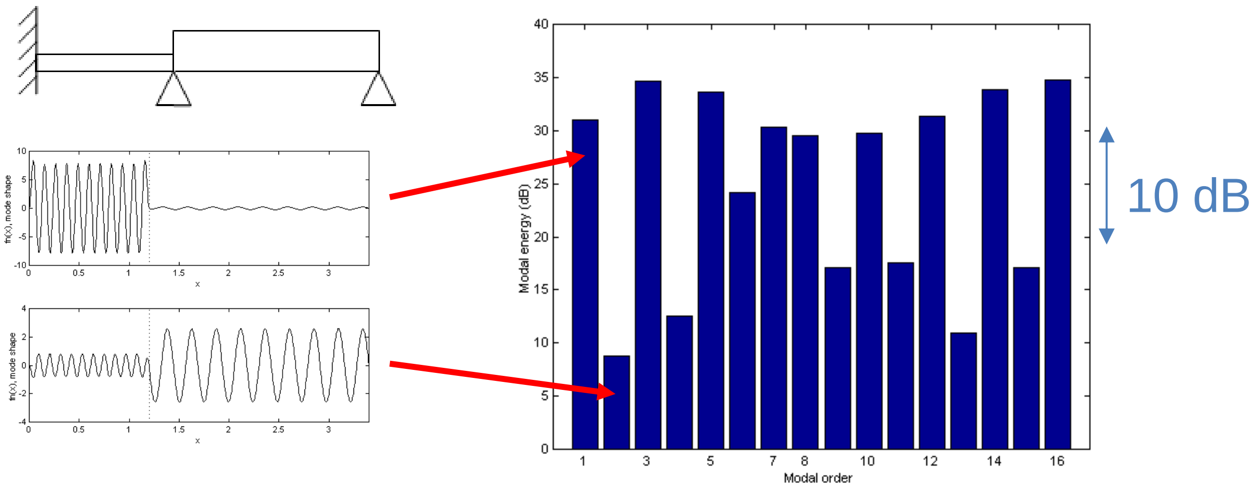


Substructuring in 3 subsystems (by a silly student!)



Method	E4/E1
FEM (ref)	-38.3 dB
SEA	-24.0 dB
SmEdA	-36.3 dB

Energy ratio E4/E1 for the 1000 Hz octave band



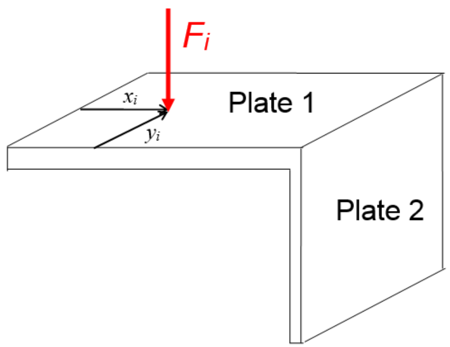
Example of two modes shapes of subsystem 2

Modal energy distribution for subsystem 2 (1000 Hz octave band)

➔ Even if a impedance mismatch has not been considered in the SmEdA substructuring, it is taken into account in the model (through the spatial mode shapes).

L. Maxit, J.L. Guyader - Extension of SEA model to subsystems with non uniform modal energy distribution. *Journal of Sound and Vibration,* 265 (2003) 337-358.

(3) Spatially localised excitations



Two coupled plates excited by a point force

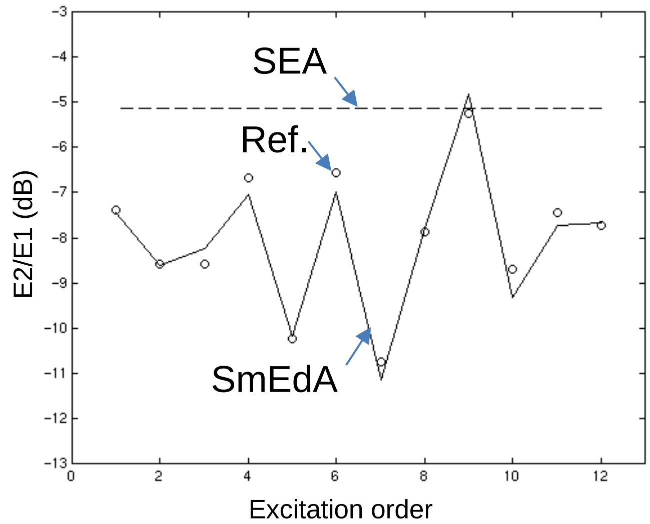
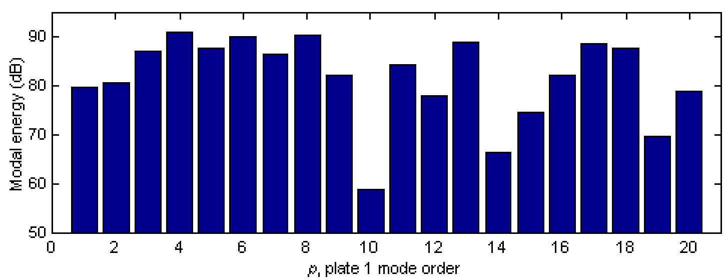
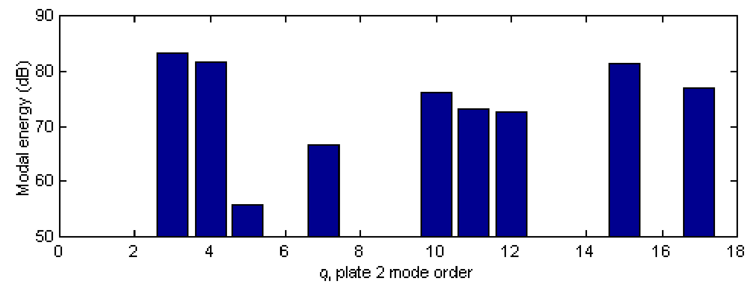


Plate energy ratio (dB) for 12 different excitation points.
1000 Hz third octave band results.



Model energy distribution of the excited plate.
Case of the excitation order 7.

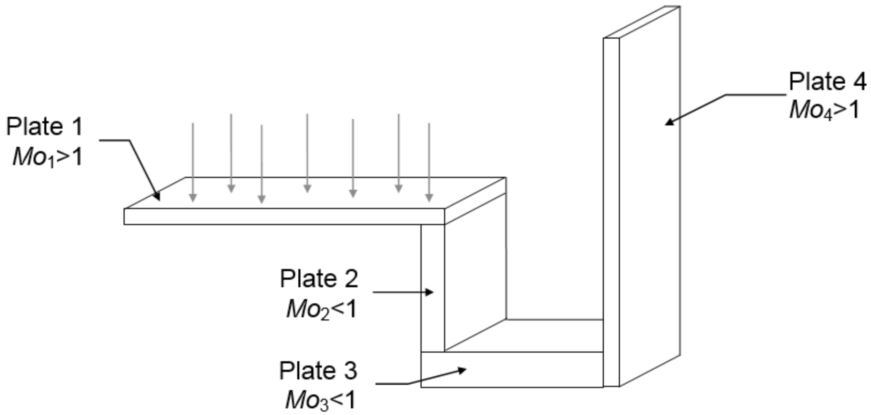


Model energy distribution of the receiving plate.
Case of the excitation order 7.

L. Maxit – Reformulation and extension of SEA model by taking the modal energy distribution into account, PhD thesis, INSA-Lyon, 2000 (in French).

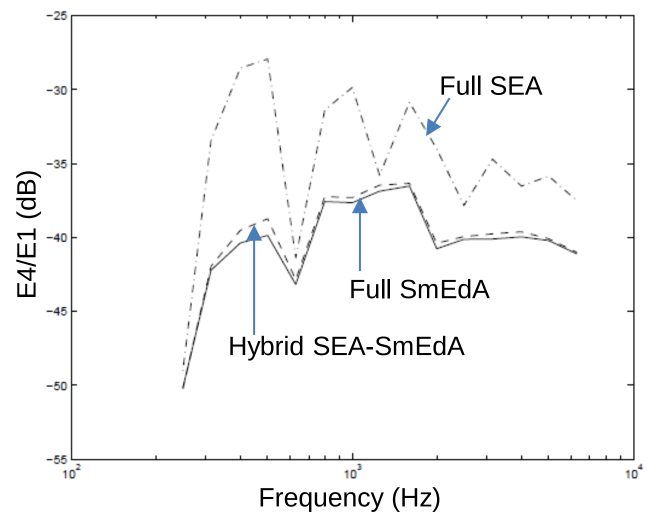
(4) Hybrid SEA/SmEdA model

Mo: Modal overlap (= Damping bandwidth x modal density).



Four coupled plates at right angle.
“Rain on the roof” excitation on plate 1.

Hybrid SEA-SmEdA model:
- plate 1 and 4 described by SEA
- plate 2 and 3 described by SmEdA

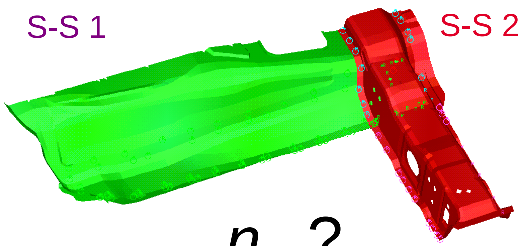


Energy ratio E4/E1 (dB) for each third octave band

L. Maxit – Reformulation and extension of SEA model by taking the modal energy distribution into account, PhD thesis, INSA-Lyon, 2000 (in French).

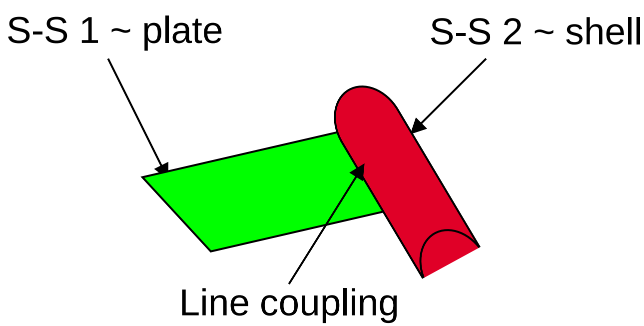
(5) Estimation of CLFs for complex subsystems

(SmEdA with equipartition assumption➔ SEA-like)



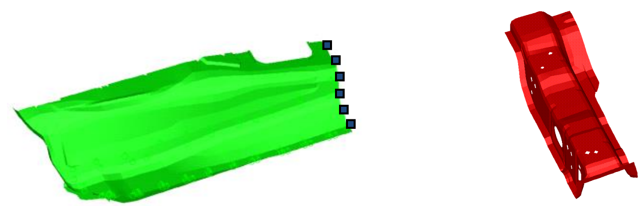
η_{12}?

« Classical-historical » approach



➔ η_{12} obtained from the travelling wave approach

SmEdA approach

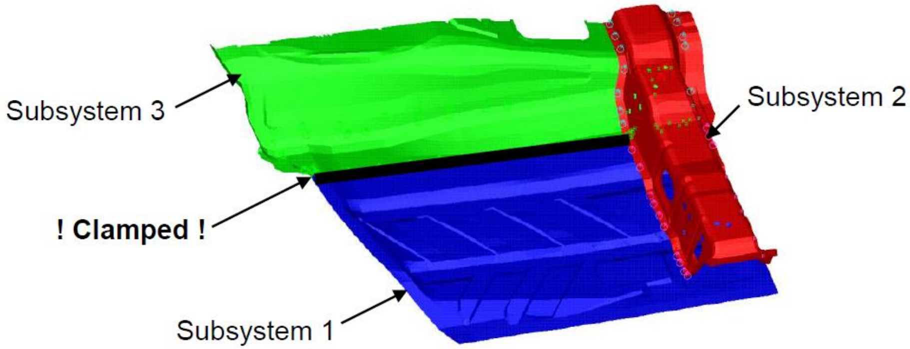


Calculation of the normal modes of each **uncoupled**-subsystem with FEM

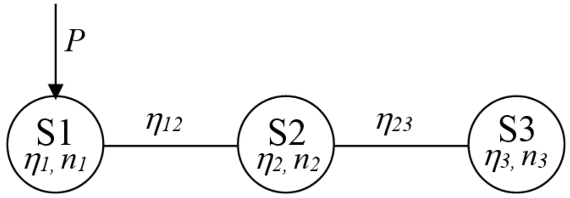
➔ η_{12} deduced from the analytical expression depending on the mode information (i.e. frequency, shape, loss factor)

L. Maxit, J.L. Guyader - Estimation of sea coupling loss factors using a dual formulation and fem modal information, part 2 : numerical applications. *Journal of Sound and Vibration,* 239 (2001) 931-948.

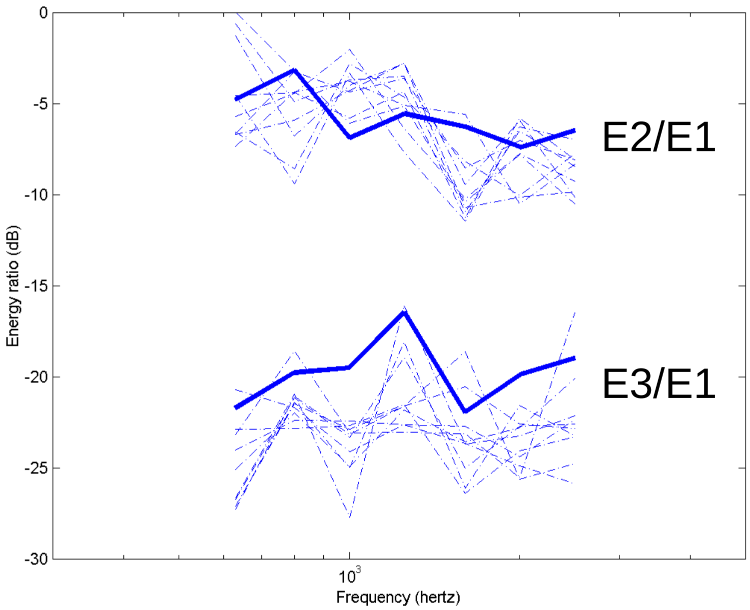
Comparison with virtual experiments (FEM simulation)



Test structure: Part of an automotive floor

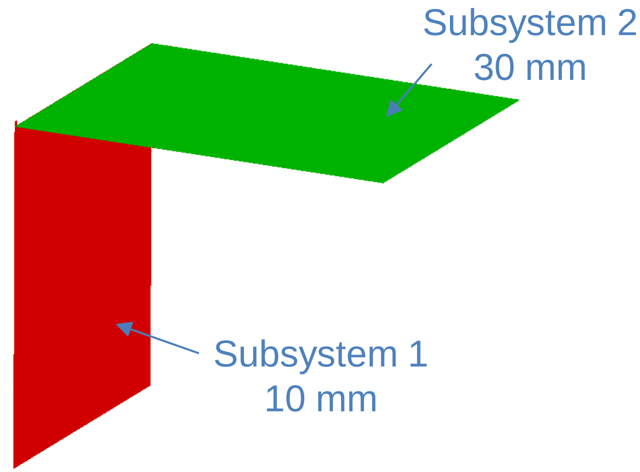


SEA model

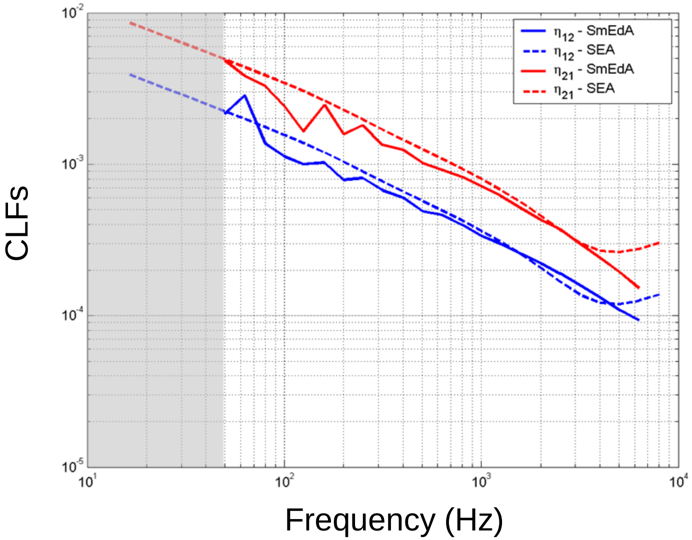


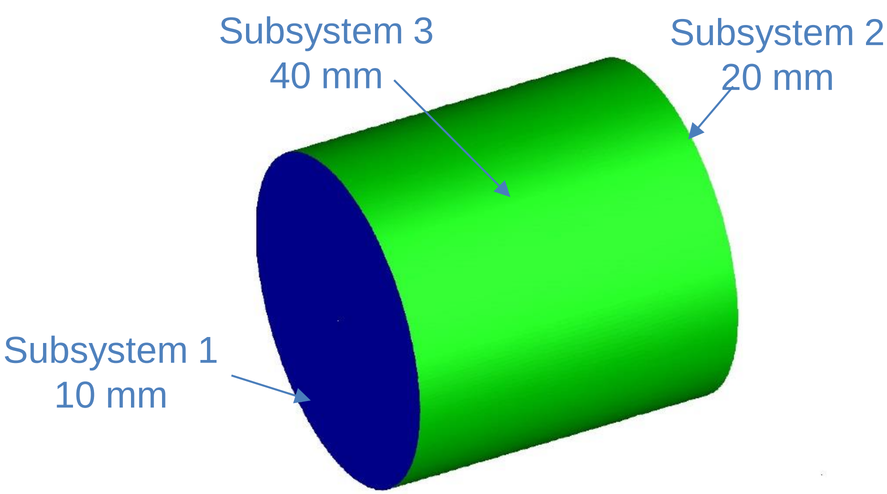
Comparison between SEA and numerical simulations for different excitation points

Comparison with the travelling wave approach on basic cases

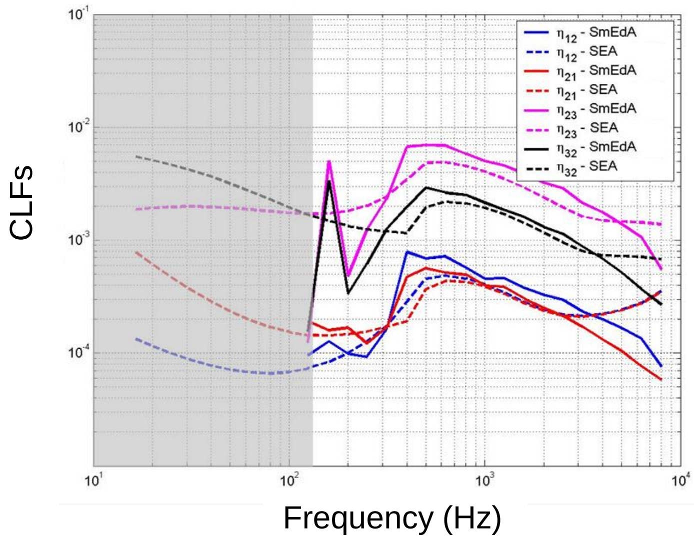


Two coupled steel panels at right angle

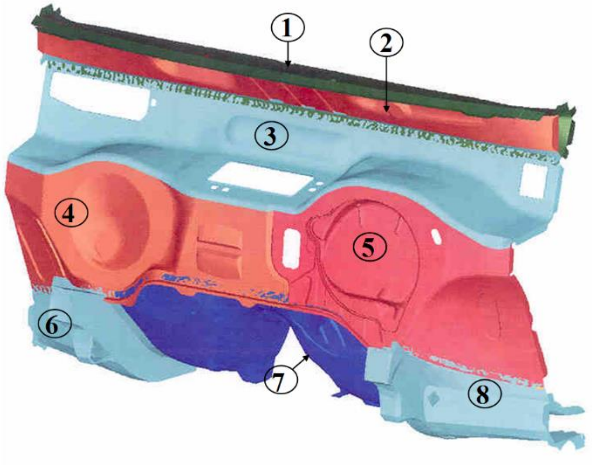




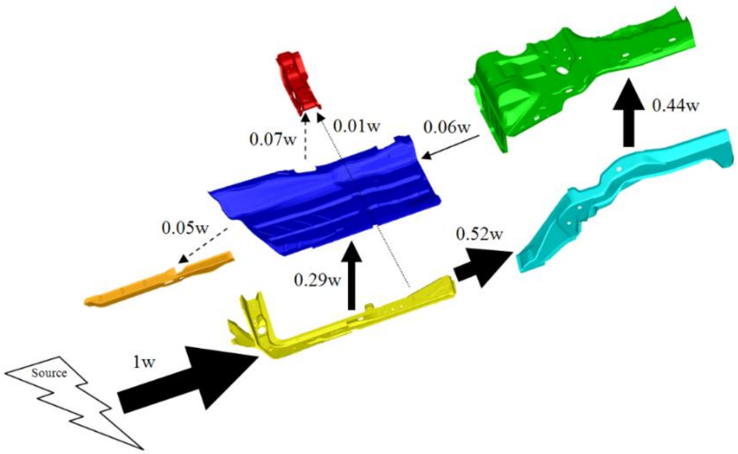
Cylindrical shell coupled to two end panels



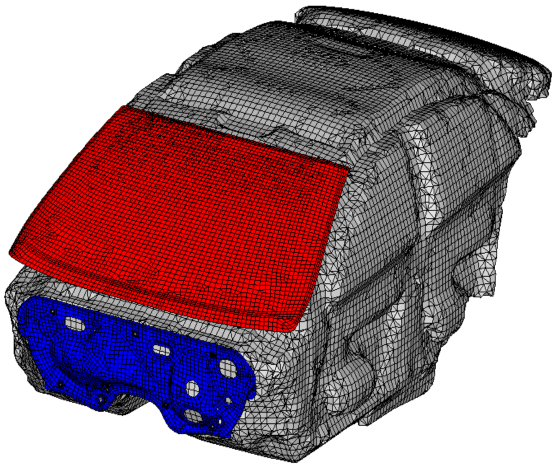
Industrial applications developped in the past



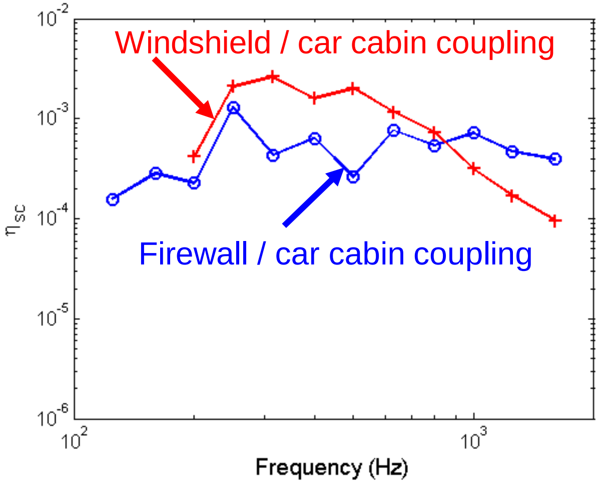
Vibration transmission through car firewall (RENAULT - 2001)



Vibration transmission through car floor (FIAT - 2002)



Sound radiation from car structure (2009)



N. Totaro, C. Dodard, J.L. Guyader, SEA Coupling Loss Factors of Complex Vibro-Acoustic Systems, JVA, ASME, 2009

(6) Estimation of the local response (spatial energy distribution)

Modal expansion for a given structural subsystem:

$$v(M,\omega)=\sum_{p=1}^{N_i} j\omega a_p(\omega)\tilde{W}_p(M)$$

Time-averaged square velocity at point M:

$$<v^2(M,\omega)>_{\Delta\omega}=\sum_{p=1}^{N_i}<\omega^2|a_p(\omega)|^2>_{\Delta\omega}\left[\tilde{W}_p(M)\right]^2+\sum_{p=1}^{N_i}\sum_{\substack{q=1\\ \neq p}}^{N_i}<\omega^4 a_p(\omega)\overline{a}_q(\omega)>_{\Delta\omega}\tilde{W}_p(M)\tilde{W}_q(M)$$

Neglected

Modal energy in function of the modal amplitude:

$$E_p\approx 2<E_{p,Kinetic}>_{\Delta\omega}=<\omega^2|a_p(\omega)|^2>_{\Delta\omega}M_p$$

Time-averaged square velocity at point M in function of the modal energies:

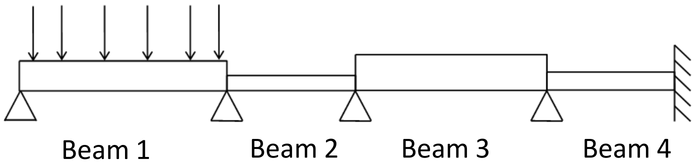
$$<v^2(M,\omega)>_{\Delta\omega}\approx\sum_{p=1}^{N_i}\frac{E_p}{M_p}\left[\tilde{W}_p(M)\right]^2$$

… similar for an acoustic subsystem.

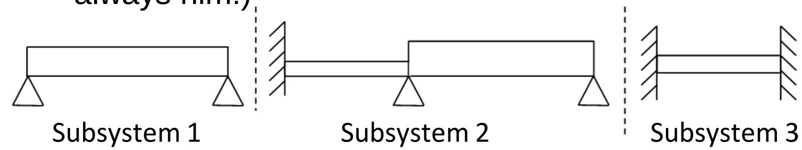
N. Totaro, J.L. Guyader - Extension of the statistical modal energy distribution analysis for estimating energy density in coupled subsystems. Journal of Sound and Vibration, 331 (2012) 3114-3129.

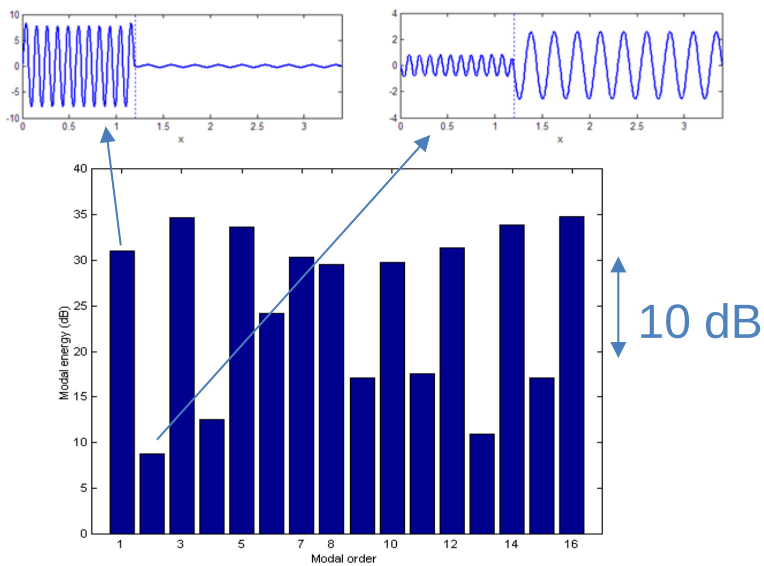
III. Interests of Statistical modal Energy distribution Analysis

Example: Four coupled beams

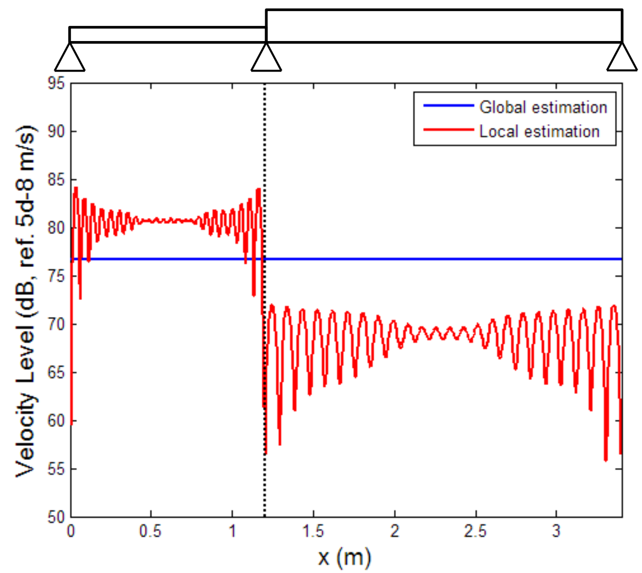


Substructuring in 3 subsystems (by a silly student!... always him!)





Modal energy distribution for subsystem 2 (1000 Hz octave band)



Estimation of the spatial variation of the velocity level for subsystem 2 using SmEdA (1000 Hz octave band)

Illustration of SmEdA application: Estimation of the pressure at the driver ear of a truck cab when the floor is excited by a point force

CLiC
City Lightweight & Innovative Cab

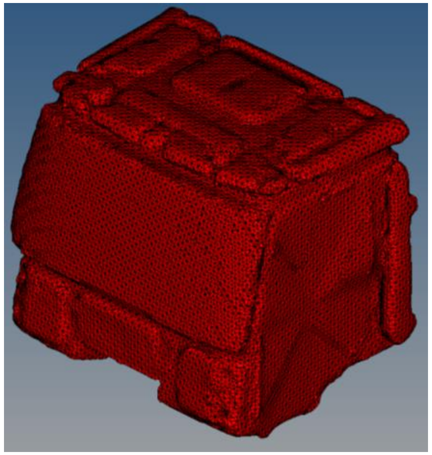
FUI-FEDER project (2012-2015)

Experimental set-up: supported truck cab excited by a mechanical force on a girder

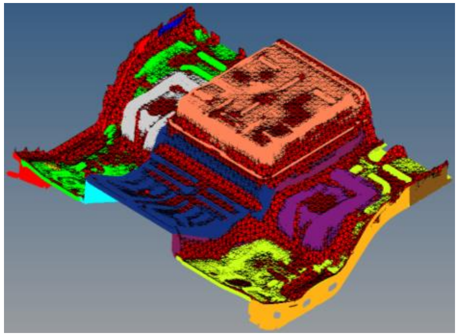
- ➔ dominant path (assumption): floor radiating into the cavity
- ➔ First studied configuration: structure body in white

Y. Gerges, et al., Mid-frequency vibroacoustic modeling of an innovative lightweight cab – floor/cavity Interaction, proceeding of VISHNO, Aix en provence, France, June 2014.

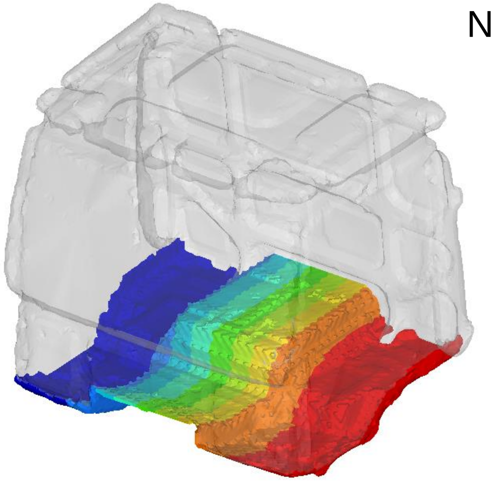
III. Interests of Statistical modal Energy distribution Analysis



FE mesh of the truck cavity

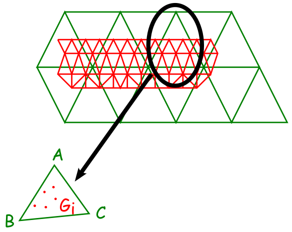


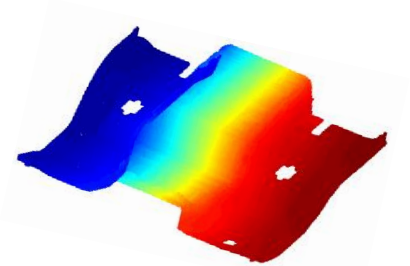
FE mesh of the floor



Example of cavity mode shape

Non-coincident meshes





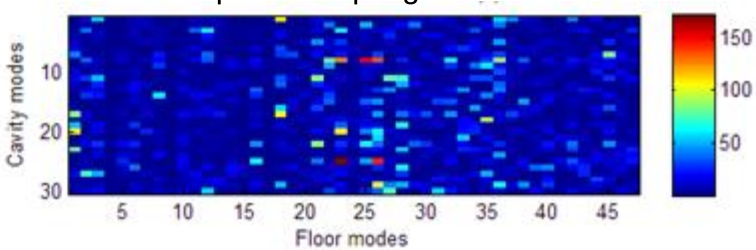
Projection of the cavity mode shape on the floor mesh

Y. Gerges, H.D. Hwang, et al., Vibroacoustic modeling of a trimmed truck cab in the mid frequency range. Proceeding of Internoise 2015, San Francisco, USA, August 2015

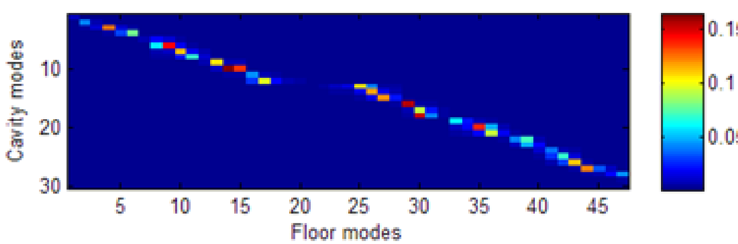
Example of intermodal coupling factors between the floor and the cavity (400 Hz third octave band):

$$\beta_{pq}^{12} = \frac{\left(W_{pq}^{12}\right)^2}{M_p^1\left(\omega_q^2\right)^2 M_q^2}\left\{\frac{\Delta_p^1\left(\omega_q^2\right)^2+\Delta_q^2\left(\omega_p^1\right)^2}{\left[\left(\omega_p^1\right)^2-\left(\omega_q^2\right)^2\right]^2+\left(\Delta_p^1+\Delta_q^2\right)\left[\Delta_p^1\left(\omega_q^2\right)^2+\Delta_q^2\left(\omega_p^1\right)^2\right]}\right\}.$$

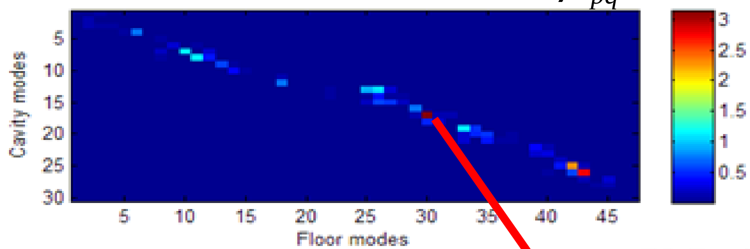
Spatial coupling factors

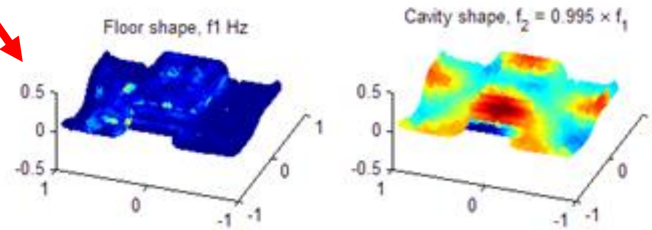


Spectral coupling factors



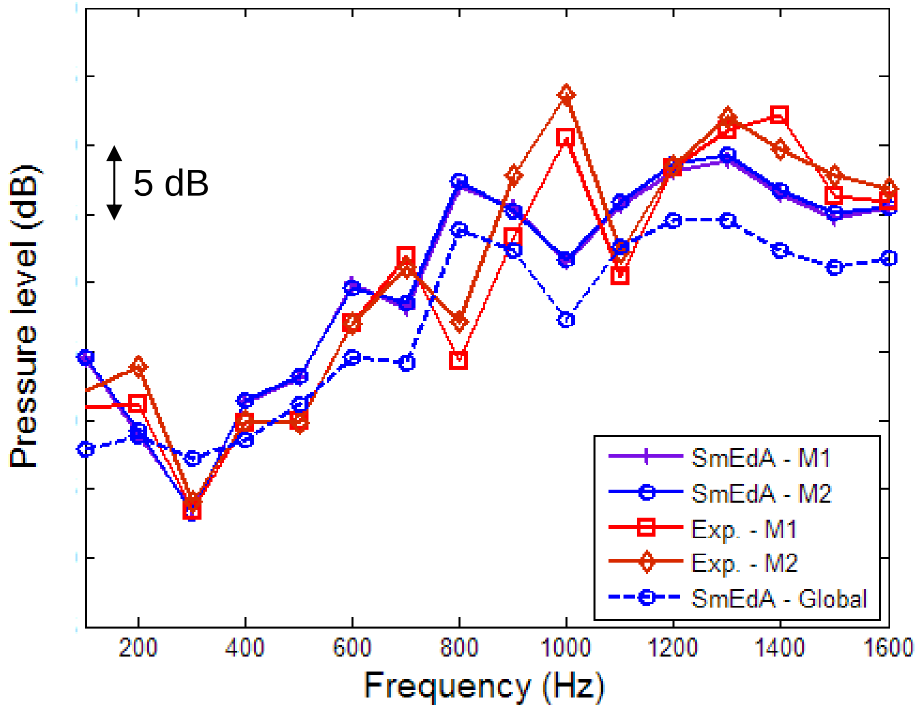
Intermodal coupling factors β_{pq}^{12}





SmEdA process for local energy estimation:

- Normal mode analysis on the uncoupled-subsystems (i.e. floor, cavity)
- Intermodal coupling factors calculations
- Modal energies estimation (by resolving the SmEdA equations)
- Local pressure estimation from the modal energies



Y. Gerges, H.D. Hwang, et al., Vibroacoustic modeling of a trimmed truck cab in the mid frequency range. Proceeding of Internoise 2015, San Francisco, USA, August 2015

SmEdA overview

- An extension of SEA by relaxing the modal energy equipartition assumption;
  - ➔ application in the mid-frequency range (low modal overlap subsystems, local excitation, heterogeneous subsystems)
  - ➔ analysis of the modal energy transfers
  - ➔ hybrid SEA/SmEdA models

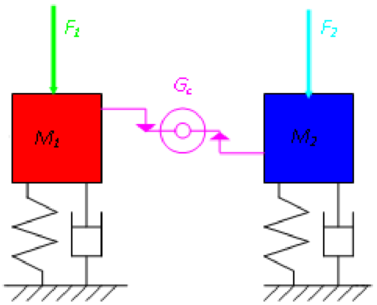
- A formulation based on the knowledge of the subsystem modes;
  - ➔ Used of subsystem Finite Element Models (FEM) for complex geometry
  - ➔ Useful tool for studying the different assumptions of the SEA (or SmEdA) fundamentals (see chapter 4, [*])

- A basic deterministic formulation
  - ➔ Extension for including non-parametric uncertainties (like SEA)
  - ➔ Framework for future developments to include different degrees of uncertainties

[*] L. Maxit – Reformulation and extension of SEA model by taking the modal energy distribution into account, PhD thesis, INSA-Lyon, 2000 (in French!).

Why there is an issue for the energy transmission between the non-resonant modes (in the SEA and SmEdA models)?

Oscillators excited by white noise force in the frequency band [0,∞[



$$\omega_1 = \sqrt{\frac{K_1}{M_1}} \qquad \omega_2 = \sqrt{\frac{K_2}{M_2}}$$

$$\Downarrow$$

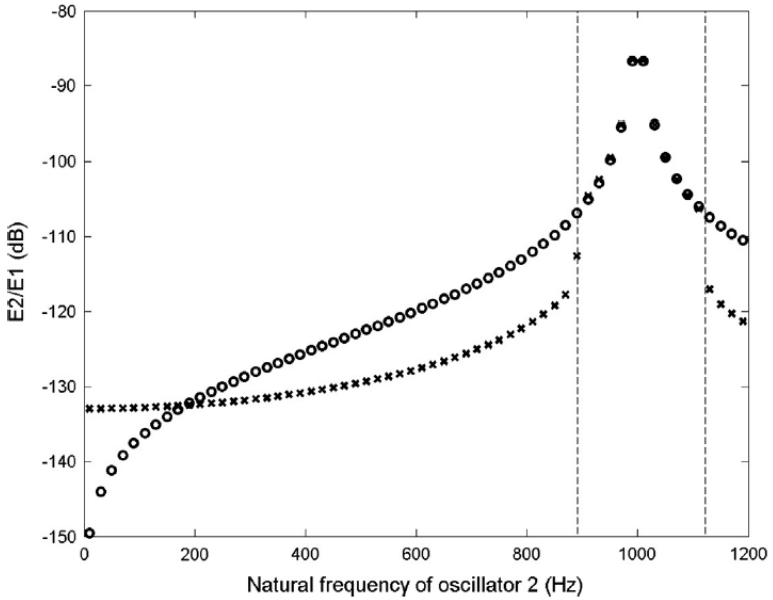
$$P_{12} = \beta\left(E_1 - E_2\right)$$

Oscillators excited by white noise force in the frequency band [Ω_1, Ω_2]

$$P_{12} \approx \beta\left(E_1 - E_2\right)$$

$$\text{if } \begin{cases} \omega_1 \in [\Omega_1, \Omega_2] \\ \omega_2 \in [\Omega_1, \Omega_2] \end{cases}$$

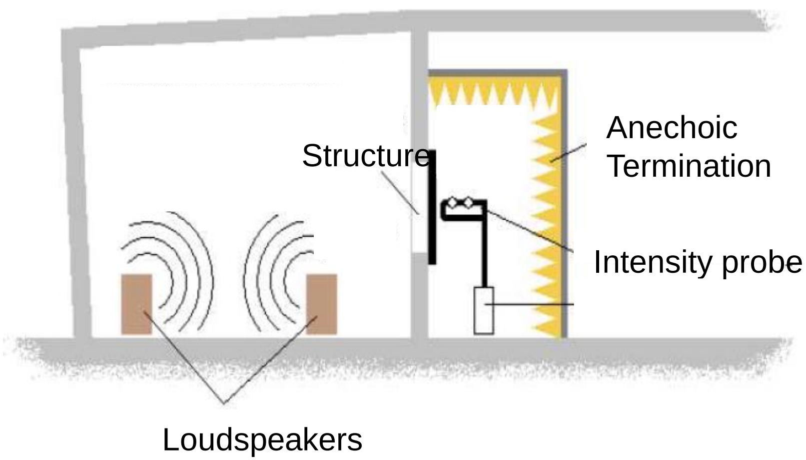
Case of oscillator 1 excited by a white noise force in the 1000 Hz third octave band



Oscillator energy ratio E2/E1 versus the natural frequency of oscillator 2. Natural frequency of oscillator 1 = 1000 Hz.

Context: Evaluation and analysis of the noise transmission through truck cab structures

Truck cab



Experimental set-up for TL evaluation

Question 1: What is the TL of the floor when the emitting cavity is the engine compartment and the receiving cavity is the truck cabin ?

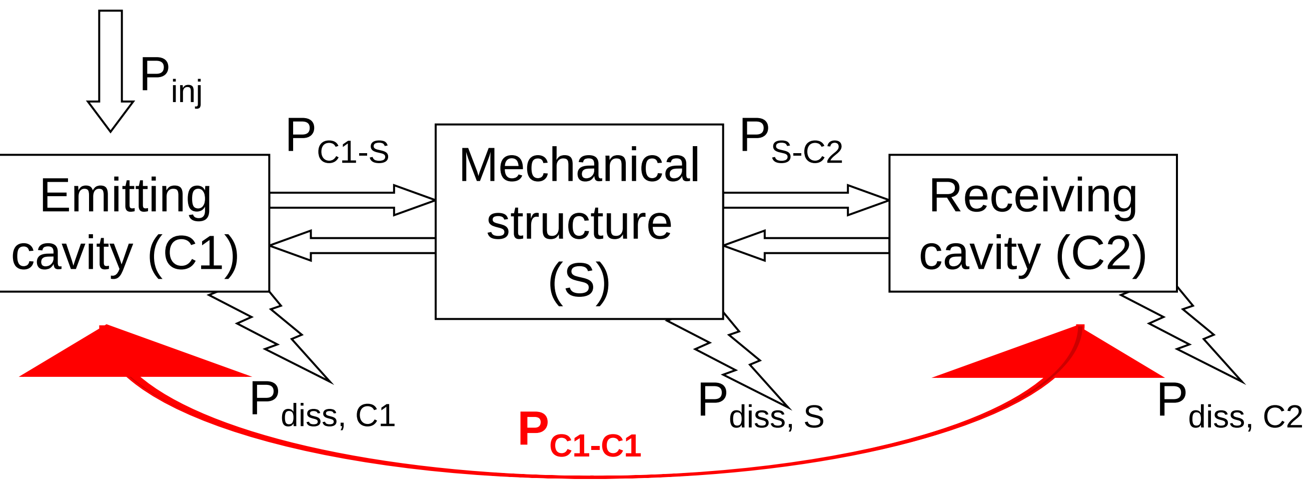
➔ Influence of the sizes and shapes of the cavities on the noise transmission

➔ Interest for a predictive method to estimate the TL of complex structures taking the behavior of small cavities and the structure geometry into account.

Estimation of the TL using Statistical Energy Analysis (SEA) model (for high frequency).

➔ For f>fc (i.e.resonant transmission), the classical SEA model can be applied.

➔ For f<fc, Crocker and Price, JSV 9 (1969) proposed a modified SEA model to take the non resonant transmission into account.



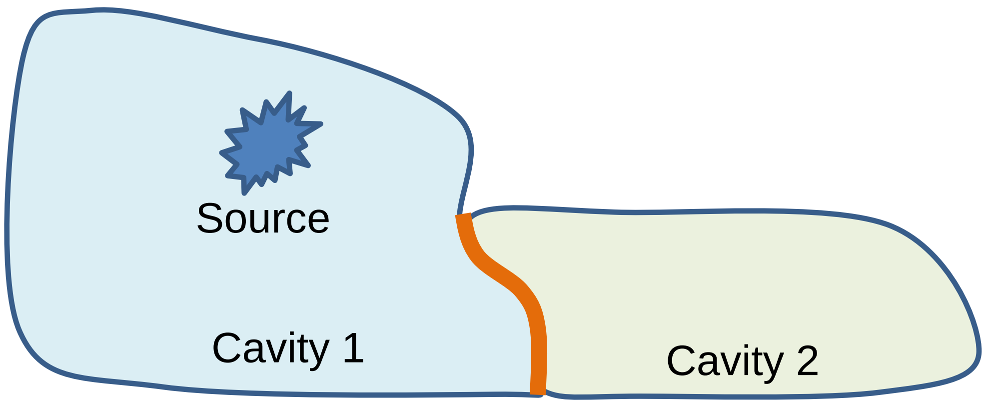
Direct coupling of the 2 cavities

➔ Mass law transmission

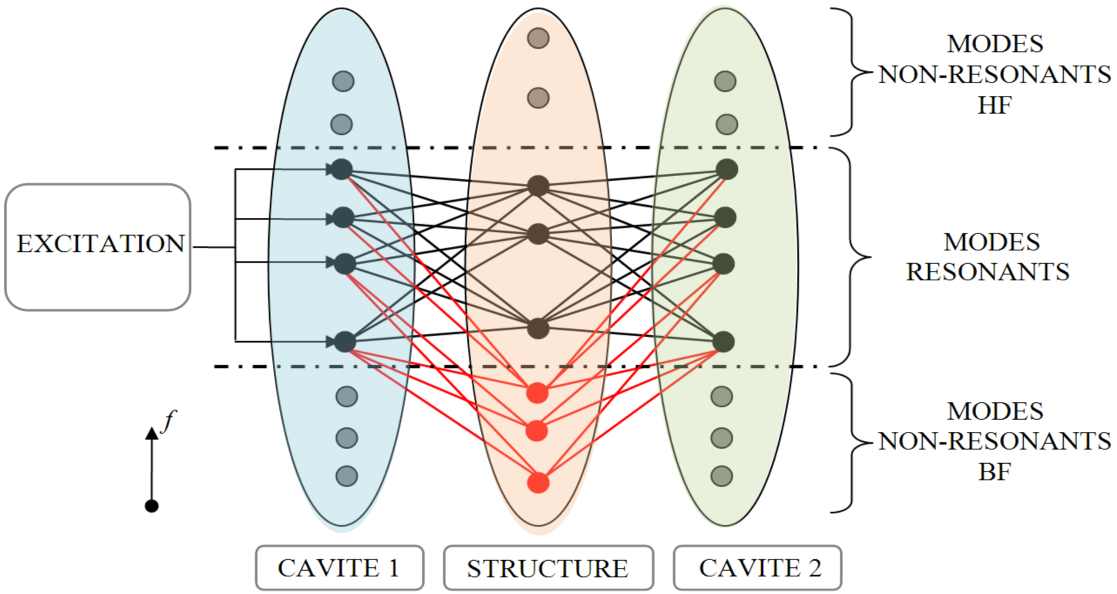
➔ CLF estimated for infinite plate (Crocker), double panel (Cray).

<u>Question 2:</u> How can we estimated this CLF for a structure with a complex geometry?

IV. Extension of SmEdA to non-resonant transmission



Transmission Loss problem for complex cavity



Modal interaction
(Black, resonant transmission, red, non resonant transmission)

The Dual Modal Formulation (DMF) allows us to write the matrix system:

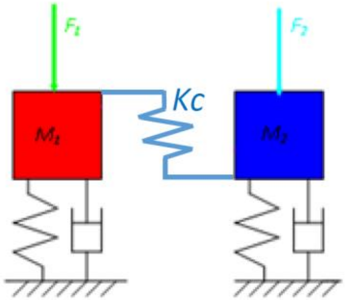
$$\begin{bmatrix} Z_{11} & -j\omega W_{12} & 0 \\ +j\omega W_{12}^{*} & Z_{22} & +j\omega W_{23}^{*} \\ 0 & -j\omega W_{23} & Z_{33} \end{bmatrix} \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \\ \Gamma_3 \end{bmatrix} = \begin{bmatrix} Q_1 \\ 0 \\ 0 \end{bmatrix}$$

Considering two sets of modes for the structure: the resonant (R) and the non-resonant (NR) gives:

$$\begin{bmatrix} Z_{11} & -j\omega W_{12}^{NR} & -j\omega W_{12}^{R} & 0 \\ +j\omega W_{12}^{NR*} & Z_{22}^{NR} & 0 & +j\omega W_{23}^{NR*} \\ +j\omega W_{12}^{R*} & 0 & Z_{22}^{R} & +j\omega W_{23}^{R*} \\ 0 & -j\omega W_{23}^{NR} & -j\omega W_{23}^{R} & Z_{33} \end{bmatrix} \begin{bmatrix} \Gamma_1 \\ \Gamma_2^{NR} \\ \Gamma_2^{R} \\ \Gamma_3 \end{bmatrix} = \begin{bmatrix} Q_1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

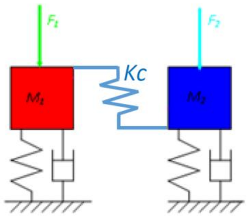
Achieving a condensation on the NR modes and assuming mass controlled behaviour for these modes, we obtain:

$$\begin{bmatrix} Z_{11} & -j\omega W_{12}^{R} & -W_{12}^{NR} W_{23}^{NR*} \\ +j\omega W_{12}^{R*} & Z_{22} & +j\omega W_{23}^{R*} \\ -W_{23}^{NR} W_{12}^{NR*} & -j\omega W_{23}^{R} & Z_{33} \end{bmatrix} \begin{bmatrix} \Gamma_1 \\ \Gamma_2^{r} \\ \Gamma_3 \end{bmatrix} = \begin{bmatrix} Q_1 \\ 0 \\ 0 \end{bmatrix}$$



L. Maxit, K. Ege, N. Totaro, J.L. Guyader - Non resonant transmission modelling with SmEdA for evaluating the transmission loss of complex structures, *Journal of Sound and Vibration, 333 (2014) 499-519.*

Direct intermodal coupling factor between the two cavities:



$$\beta_{pr} = \left(\sum_{q \in Q^{NR}} W_{pq} W_{rq} \right)^2 \left\{ \frac{\left(\omega_p \eta_p + \omega_r \eta_r\right)}{\left[\left(\omega_p\right)^2 - \left(\omega_r\right)^2\right]^2 + \left(\omega_p \eta_p + \omega_r \eta_r\right)\left[\omega_p \eta_p \left(\omega_r\right)^2 + \omega_r \eta_r \left(\omega_p\right)^2\right]} \right\}.$$

p : modes of cavity 1
r : modes of cavity 2
q : non-resonant modes of the plate

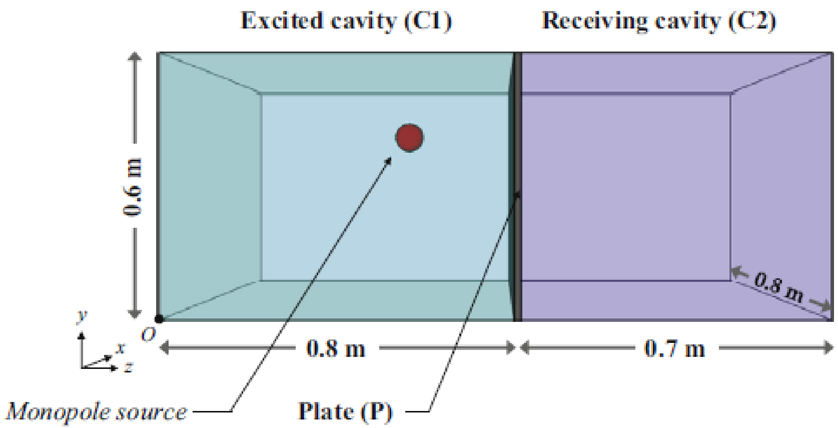
W_{pq} W_{rq} are the intermodal works between modes of the cavities and non-resonant modes of the structure

SEA coupling loss factor between the 2 cavities:

$$\eta_{C1-C2} = \frac{\sum_{p \in P} \sum_{q \in R} \beta_{pr}}{P \omega_c}$$

L. Maxit, K. Ege, N. Totaro, J.L. Guyader - Non resonant transmission modelling with SmEdA for evaluating the transmission loss of complex structures, *Journal of Sound and Vibration, 333 (2014) 499-519.*

Acoustic transmission between two “Small” cavities



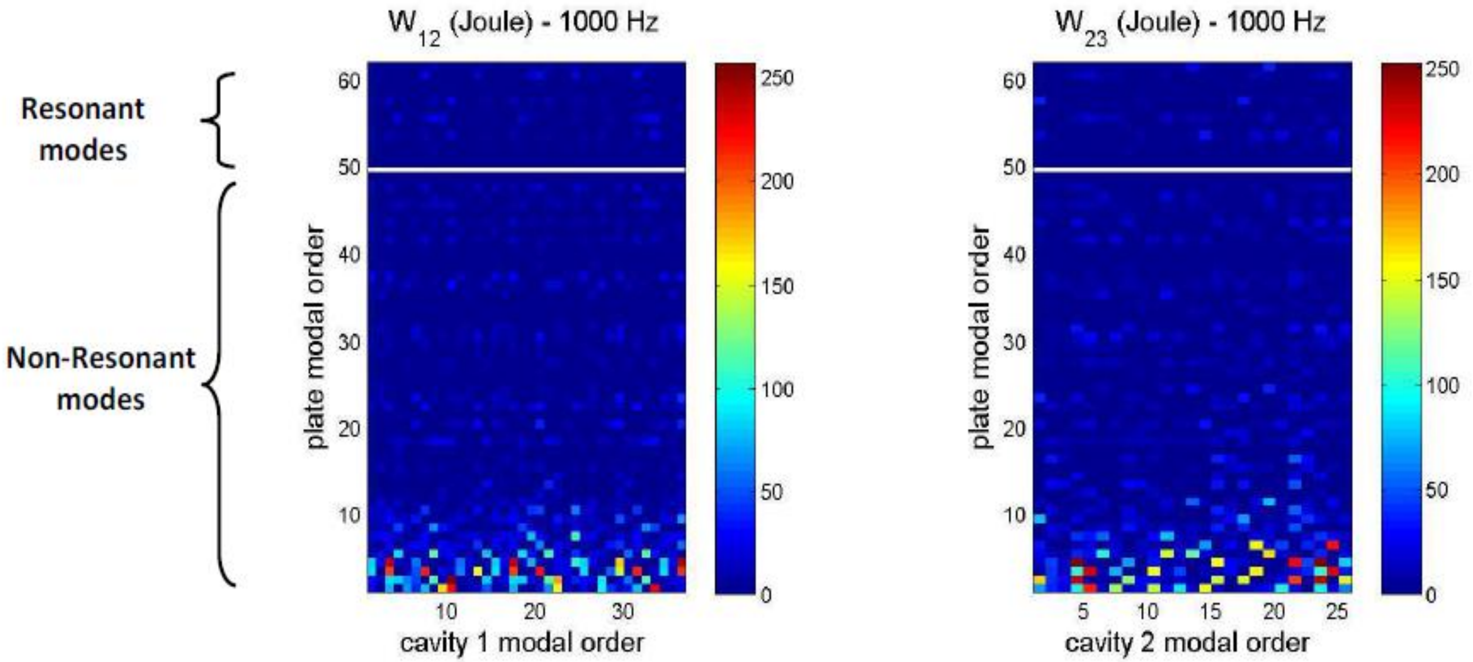
	400 Hz	500 Hz	630 Hz	800 Hz	1 kHz	1.25kHz	1.6 kHz	2 kHz	2.5 kHz
P	5	6	12	22	41	71	149	263	535
Q^{NR}	46	59	75	96	124	157	198	251	322
Q^{R}	13	16	21	28	33	42	52	70	83
R	4	4	12	21	32	67	129	231	472

	3.15kHz	4 kHz	5 kHz	6.3 kHz	8kHz	10kHz	12.5kHz	16kHz	20kHz
P	1033	1998	3982	7815	15490	30672	60818	121228	236518
Q^{NR}	406	515	655	829	1049	1331	1682	2122	2687
Q^{R}	108	139	173	219	281	350	439	564	703
R	909	1766	3483	6859	13560	26876	53361	106085	209151

Subsystem mode numbers for each third octave band

IV. Extension of SmEdA to non-resonant transmission

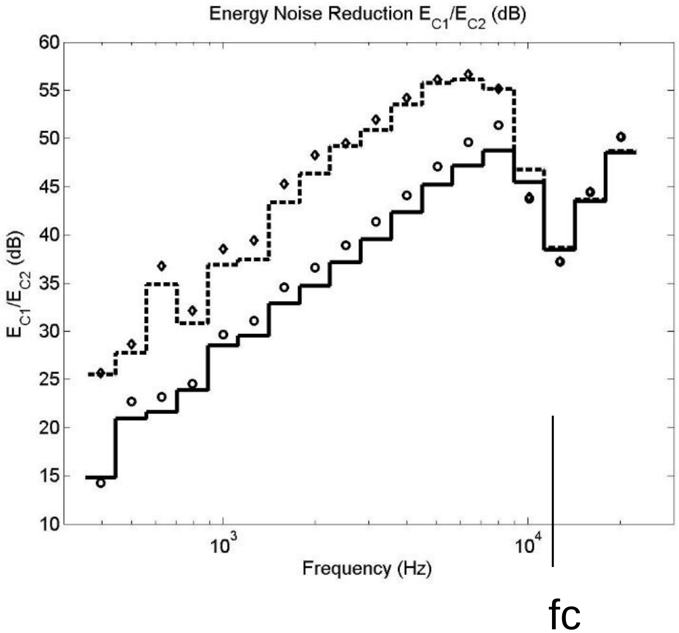
$$W_{pq} = \int_S \tilde{W}_q \tilde{p}_p dS \qquad W_{qr} = \int_S \tilde{W}_q \tilde{p}_r dS$$



Intermodal works (Third octave band 1000 Hz)

1mm-thick steel plate (critical frequency ~ 11.7 kHz)

Energy Noise Reduction versus third octave band:



Comparison of four calculations:

solid line, reference;

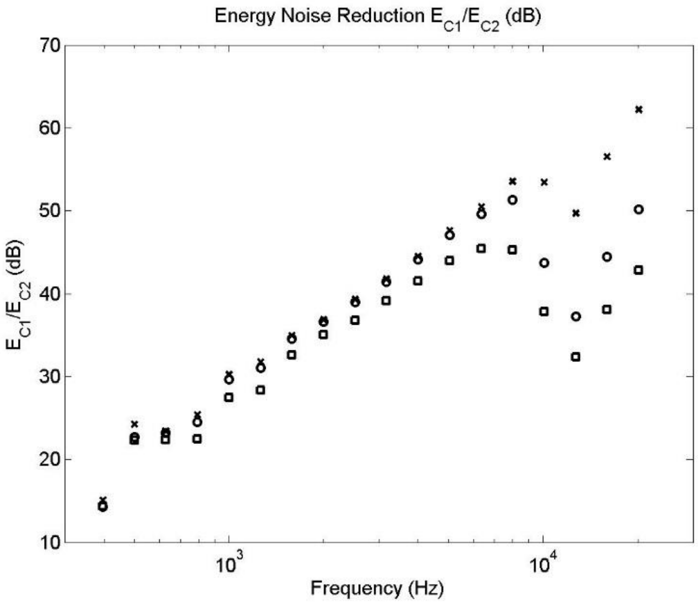
circles, SmEdA taking the NR plate modes into accoun

dashed line, DMF without NR plate modes;

diamonds, SmEdA without NR plate modes.

SmEdA results for different plate DLF:

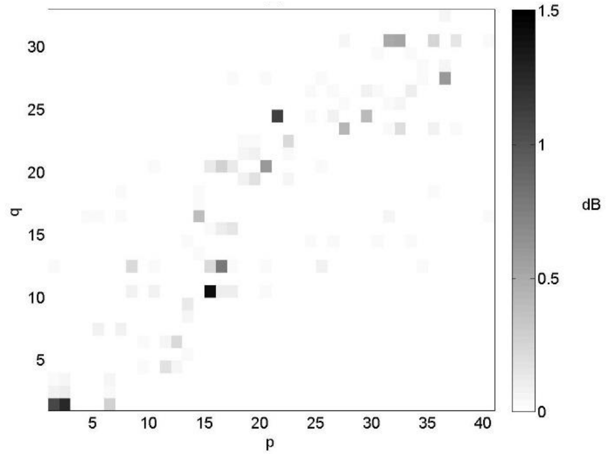
cross 10%; circles, 1% ; square, 0.1%.



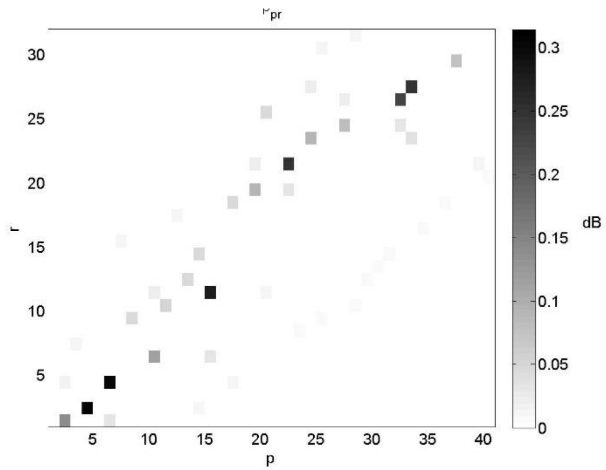
IV. Extension of SmEdA to non-resonant transmission

Modal transfer path analysis

Emitting cavity – panel (Resonant transmission)

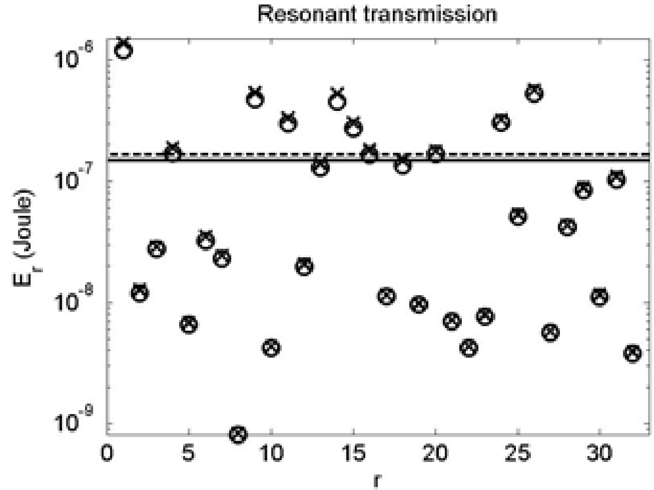


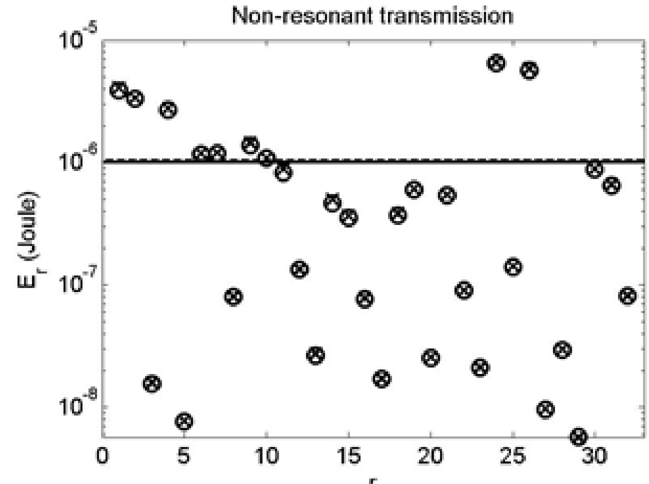
Emitting cavity – Receiving cavity (NR transmission)



Modal coupling loss factors
- 1000 Hz third octave

x, SmEdA; o, Simplified SmEdA

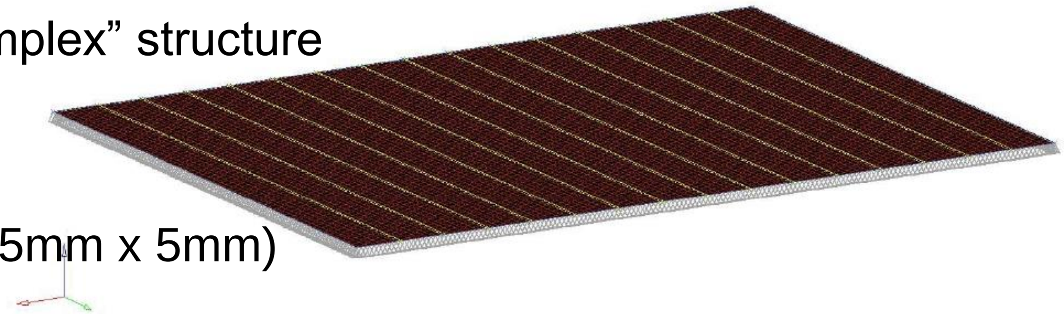




Modal energy distribution for the receiving cavity - 1000 Hz third octave

Example of application for "complex" structure

➔Ribbed plate
Plate thickness: 1mm
Rib cross-section: square (5mm x 5mm)
Rib spacing: 50 mm



Finite element meshing of the ribbed plate
19481 Nodes, 19200 CQUAD4, 1800 CBEAM

↓

Natural frequencies and mode shapes calculation until 10 kHz (MD NASTRAN)

↓

Analytical cavity modes ➔ Intermodal works (integral: rectangular rule)

↓

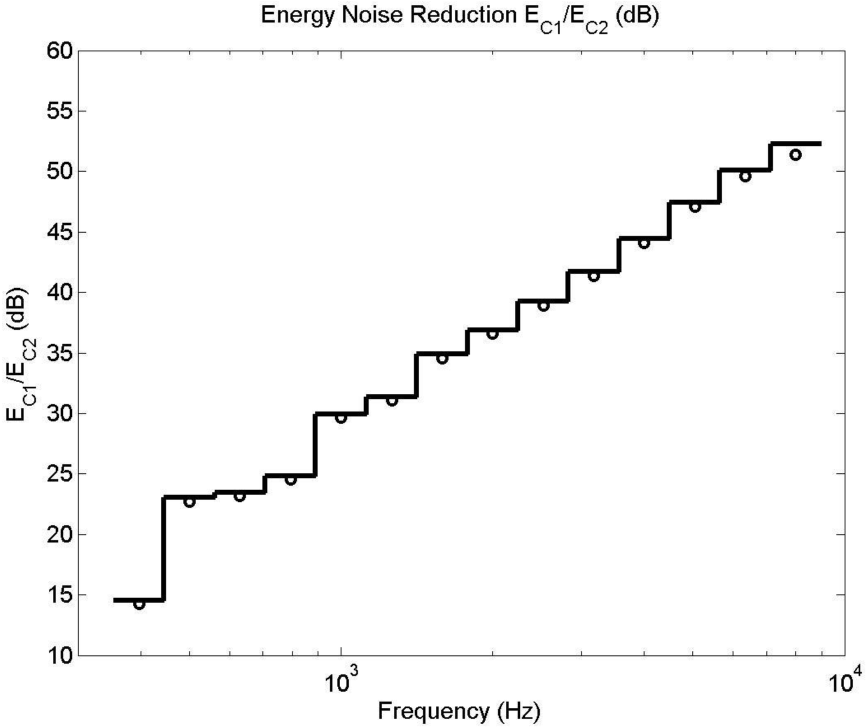
SmEdA calculation by third octave band

↓

Transmission Loss

Validation of the SmEdA process with FEM modes

FEM mesh size criterion: flexural wavelength / 6
➔ 5 % difference between the analytical and FEM modal frequencies



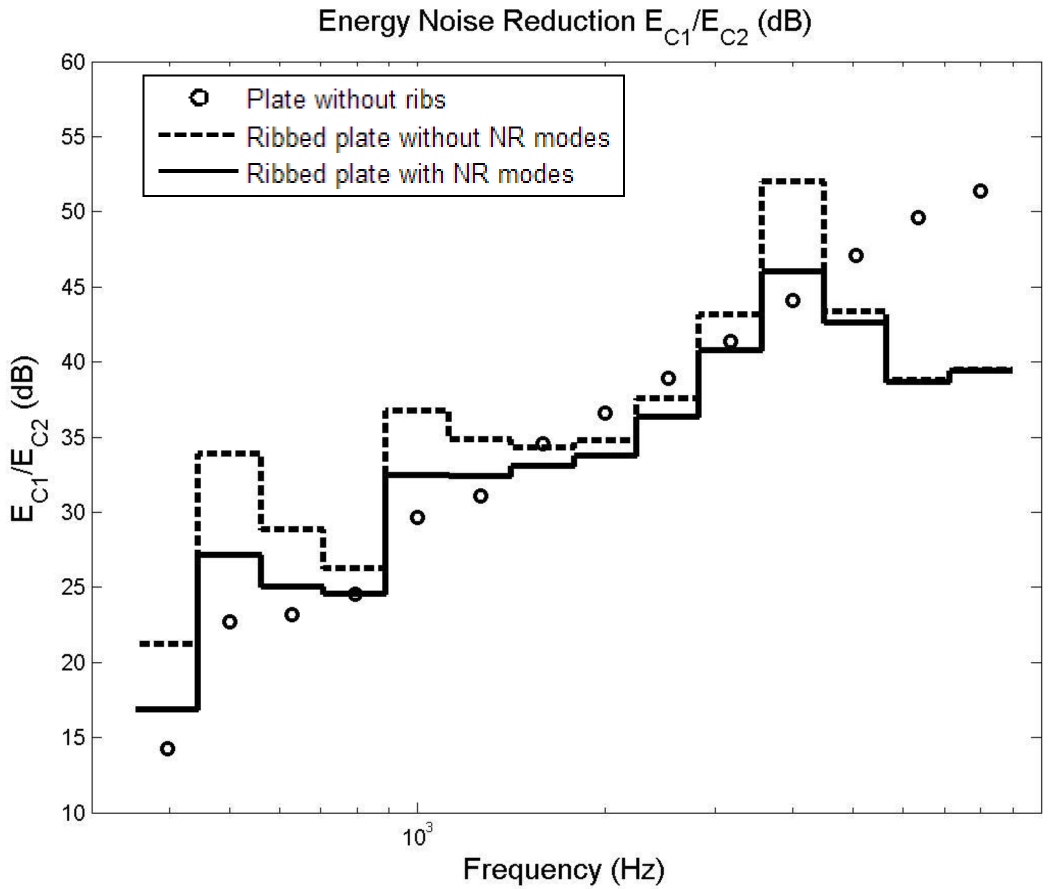
Comparison of the SmEdA results with plate modes
calculated analytically: circle; solid line, numerically with FEM

Example of results for stiffened plate:

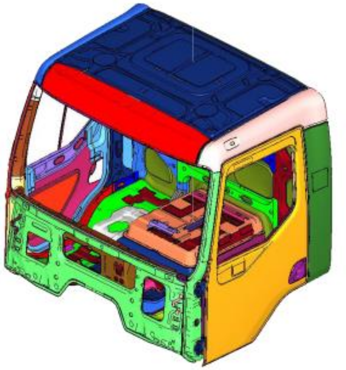
Plate thickness: 1mm

Rib cross-section: square (5mm x 5mm)

Rib spacing: 50 mm



V. Methodology for including the effect of dissipative treatments in SmEdA



Vibro-acoustic modelling of the truck cab including **dissipative treatments**

- Viscoelastic layers (Damping layer)
- Acoustic absorbing materials (trim, foarm)

➔ How to take the dissipative effect of these materials into account with a SmEdA model?

Modal energy equations of motion for two subsystems (SmEdA):

$$\begin{cases} \Pi_{inj}^{1p} = \left(\omega_p^1 \eta_p^1 + \sum_{q'=1}^{N_2} \beta_{pq'}^{12} \right) E_p^1 - \sum_{q'=1}^{N_2} \beta_{pq'}^{12} E_{q'}^2, & \forall p \in [1,\ldots,N_1], \\ \Pi_{inj}^{2q} = - \sum_{p'=1}^{N_1} \beta_{p'q}^{12} E_{p'}^1 + \left(\omega_q^2 \eta_q^2 + \sum_{p'=1}^{N_1} \beta_{p'q}^{12} \right) E_q^2, & \forall q \in [1,\ldots,N_2]. \end{cases}$$

with the intermodal coupling factors:

$$\beta_{pq}^{12} = \frac{W_{pq}^{12}}{M_p^1 (\omega_q^2)^2 M_q^2} \left\{ \frac{\omega_p^1 \eta_p^1 (\omega_q^2)^2 + \omega_q^2 \eta_q^2 (\omega_p^1)^2}{\left[(\omega_p^1)^2 - (\omega_q^2)^2 \right]^2 + \left(\omega_p^1 \eta_p^1 + \omega_q^2 \eta_q^2 \right) \left[\omega_p^1 \eta_p^1 (\omega_q^2)^2 + \omega_q^2 \eta_q^2 (\omega_p^1)^2 \right]} \right\}$$

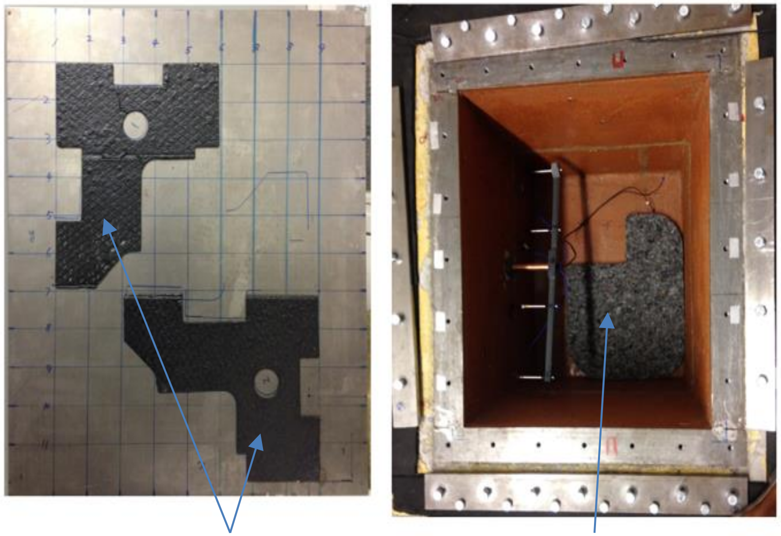
Dissipative effect ➔ Modal damping loss factors ➔ | Dissipative powers / Intermodal coupling factors

$\eta_p^1, \forall p \in [1,\ldots,N_1]$

$\eta_q^2, \forall q \in [1,\ldots,N_2]$

V. Methodology for including the effect of dissipative treatments in SmEdA

Illustration of the methodology on the validation test cases
PhD thesis H.D. HWANG (2011-2015)



Experimental set-up of the validation case
(Rectangular “clamped” plate radiating into a rectangular cavity with “rigid” wall)

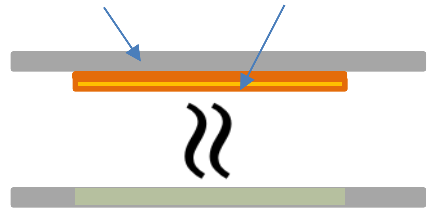
H.D. Hwang et al.- A methodology for including the effect of a damping treatment in the mid-frequency domain using SmEdA method, *Proceedings of 20th International Congress on Sound and Vibration*, Bangkok, Thailand, July 2013. 8p.

V. Methodology for including the effect of dissipative treatments in SmEdA

Illustration of the methodology on the validation test case

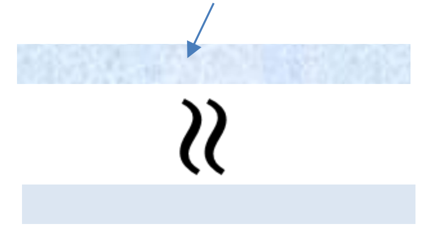
<u>1st step:</u>
Characterisation of the equivalent dissipative material

Structural subsystem



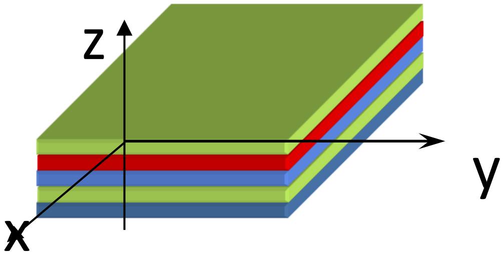
Equivalent single layer (ρ_{eq}, $E_{eq}(f)$, $\eta_{eq}(f)$)
(MOVISAND software)

Acoustic subsystem



Equivalent Fluide ($\rho_{eq}(f)$, $c_{eq}(f)$, $\eta_{eq}(f)$)
(Acoustic tube with 2 cavities method)

Equivalent Single Layer Model for Viscoelastic Materials



$$-\omega^2(\mathfrak{T})\begin{Bmatrix} W\\ \psi_x^1\\ \psi_y^1\\ \phi_x^1\\ \phi_y^1\end{Bmatrix} + (\mathfrak{V})\begin{Bmatrix} W\\ \psi_x^1\\ \psi_y^1\\ \phi_x^1\\ \phi_y^1\end{Bmatrix} = \{0\}$$

Transverse
Membrane
Shear

J.-L. Guyader, C. Lesueur. *JSV, 58(1):51–58, 1978*

- Continuity condition → function of the 1st layer
- Isotropic material → wave in x direction
- Small thickness → rotationary inertia neglected

Reduced multi-layer model: $-\omega^2\delta_{13}\bar{W} + \left\{A_1k^4 - \dfrac{A_2}{A_3k^2 + \omega\sqrt{M_s}A_4}k^6\right\}\bar{W} = 0$

Love-Kirchhoff model: $-\omega^2 M_s \bar{W}_{LK} + Bk^4\bar{W}_{LK} = 0$

δ_{13} & $A_1 \sim A_4$ depend on ω and layer material properties

One root associated to the dominant transverse motion → bending stiffness of the equivalent layer

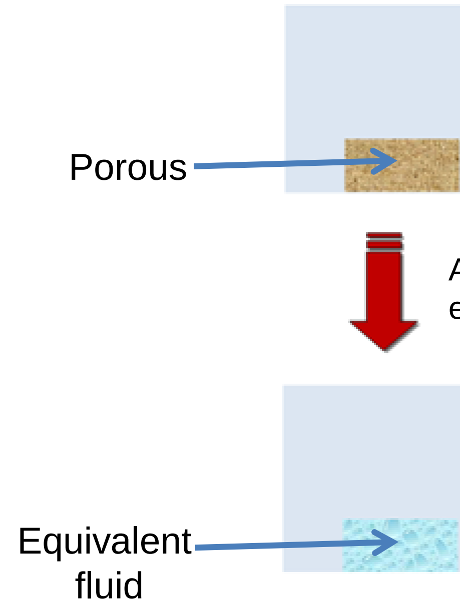
$$E_{eq}^* = \frac{12(1-v_{eq}^2)B}{h_1^3}$$

- Reduced FEM computation compared to 3D model
- Method implemented in *MOVISAND* software

Equivalent Fluid for Porous Materials

Biot's model (Theory of poroelasticity)

- Solid (u^s) + fluid (u^f)
- One shear, two compression waves
- Macroscopic properties ($\phi, \sigma, \alpha, \Lambda, \Lambda'$)

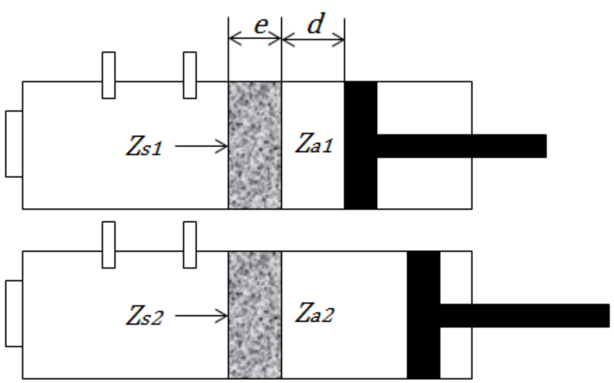


Airborne excitation ($u^s = 0$)

$$\frac{\Delta p}{\tilde{\rho}_{eq}} - \omega^2 \frac{p}{\tilde{K}_f} = 0$$

Equivalent fluid model

- Rigid solid assumption ($u^s = 0$)
- Material fixed to a rigid surface
- Characterized as a fluid (c_{eq}, ρ_{eq})
- Kundt tube measurement → Z_c & k_{eq}



Acoustic tube measurement with the two cavities method, H. Utsuno, et al. ***JASA***,86(2), 1989

$$\tilde{K}_f = \frac{Z_c \omega}{k_{eq}}$$

$$\tilde{\rho}_{eq} = \frac{Z_c^2}{\tilde{K}_f}$$

Complex quantities

→

$$\chi_e = \frac{Im\{\tilde{\rho}_{eq}\}}{Re\{\tilde{\rho}_{eq}\}}$$

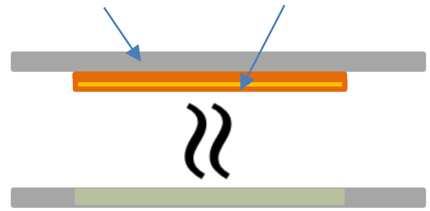
$$\eta_e = \frac{Im\{\tilde{K}_f\}}{Re\{\tilde{K}_f\}}$$

Two damping factors!.. for the viscous and thermal dissipative effect

Illustration of the methodology on the validation test case

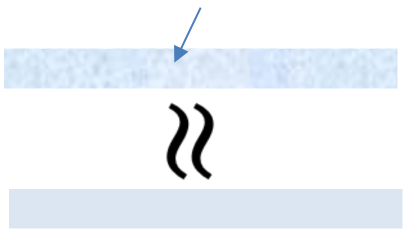
1st step:
Characterisation of the equivalent dissipative material

Structural subsystem



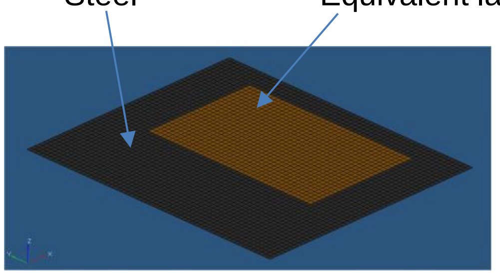
Equivalent single layer (ρ_{eq}, $E_{eq}(f)$, $\eta_{eq}(f)$)
(MOVISAND software)

Acoustic subsystem

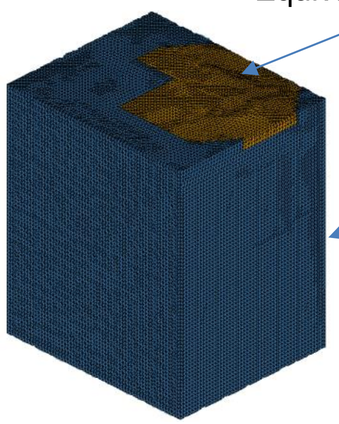


Equivalent Fluide ($\rho_{eq}(f)$, $c_{eq}(f)$, $\eta_{eq}(f)$)
(Acoustic tube with 2 cavities method)

2nd step:
Creation of FE model including the equivalent dissipative elements



FEM of the structural subsystem



FEM of the acoustic subsystem

!!! Equivalent material properties depend on the third octave band !!!

Illustration of the methodology on the validation test case

<u>3nd step:</u>
Normal modes extraction and evaluation of the modal damping loss factors (from the imaginary part of the FE matrices)
NASTRAN DMAP

$$(\omega_p, M_p, \mathbf{W}_p) \qquad (\omega_q, M_q, \mathbf{p}_q)$$

MSE method **MSKE method**

$$\eta_p = \frac{\mathbf{Im}\{\mathbf{W}_p \bar{\mathbf{K}} \mathbf{W}_p\}}{\mathbf{Re}\{\mathbf{W}_p \bar{\mathbf{K}} \mathbf{W}_p\}} \qquad \eta_q = \dots.$$

(see [PhD Hwang])

<u>4th step:</u>
Calculation of the modal coupling loss factors (MCLFs)

$$\beta_{pq} = f(\omega_p, M_p, \mathbf{W}_p, \eta_p, \omega_q, M_q, \mathbf{p}_q, \eta_q)$$

<u>5th step:</u>
Calculation of the modal energies and the subsystem energy from SmEdA equations

$$E_p, E_q$$

$$E_{t1}, E_{t2}$$

H.D. Hwang, Extension of the SmEdA method by taking into account dissipative materials in medium frequency, PhD thesis, INSA Lyon, France, 2015.

Four test cases:

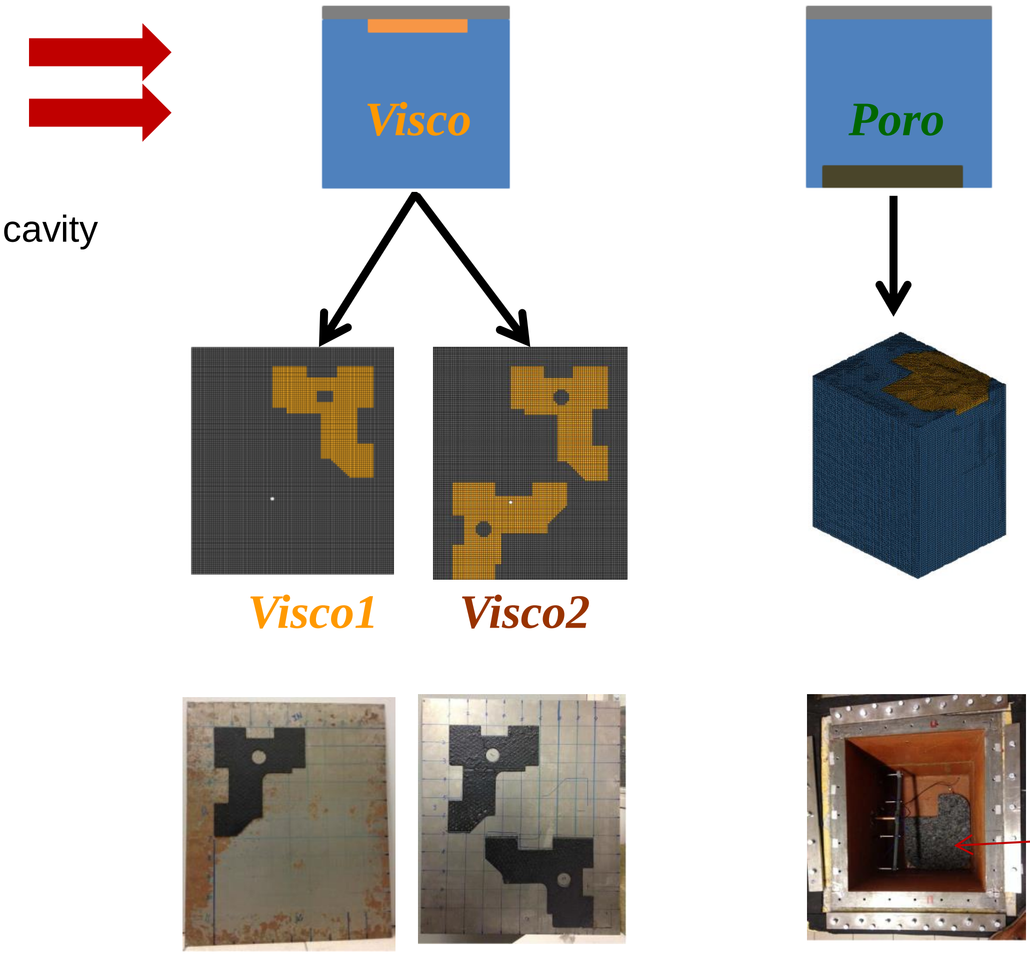
➔ Comparison of the no-treatment case to the treatment cases

➔ To study the influence of the treatments on the energy transmission

Steel plate + Acoustic cavity

Plate: 0.5*0.6*0.001 (m^2)

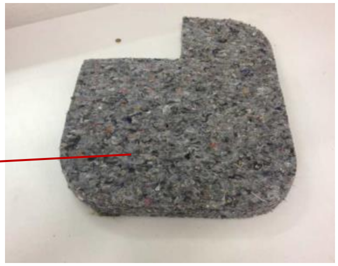
Cavity: 0.5*0.6*0.7 (m^2)



Fibrous material

•3 cm thick

•2% of the cavity volume



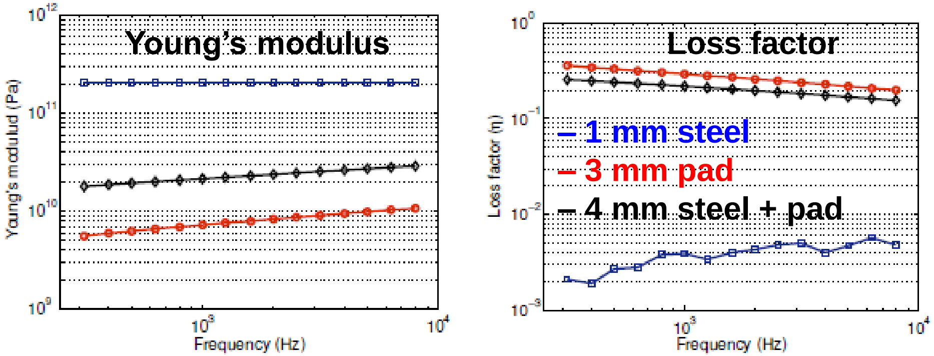
1st STEP

Characterizing
the dissipative materials
by the equivalent models

Viscoelastic damping pad

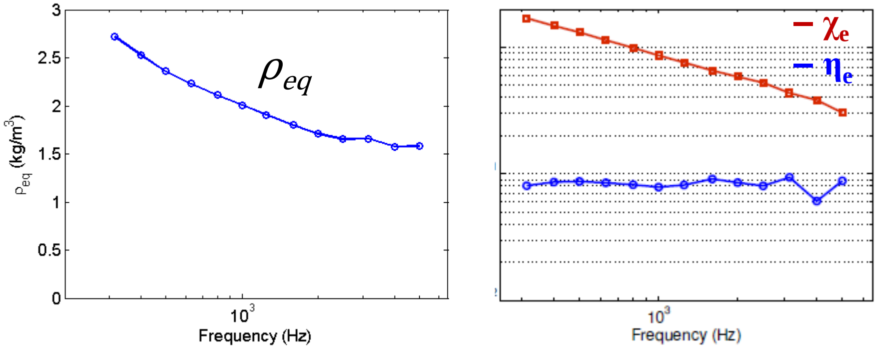
Composite fibre

Steel & Pad & Steel + Pad (Eqv) properties



1/3 octave band

Equivalent fluid properties



1/3 octave band

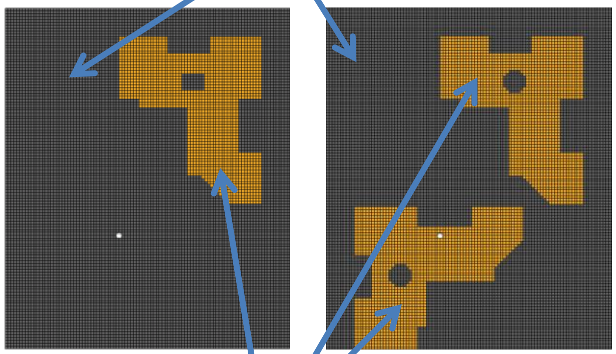
MOVISAND computation 20 ºC
Viscoelastic damping pad properties
provided by *ACOEM (Dynamic Material Analyser)*

Acoustic tube measurement with the two cavities method, Third octave band averaged.

2nd STEP

FE modeling of the subsystems with equivalent parameters

Steel: $E, \nu, \rho, \eta_{eq}(f)$

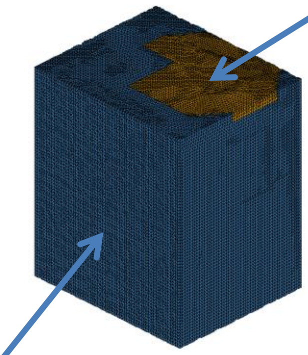


Visco1 *Visco2*

Equivalent:
$E_{eq}(f), \nu_{eq}, \rho_{eq}, \eta_{eq}(f)$

- 13,776 shell elements
- 8 kHz band
- Clamped boundaries

Equivalent:
$c_{eq}(f), \ \rho_{eq}(f)$



Air:
c_0, ρ_0

Poro

- 4,031,412 solid elements
- 6 kHz band
- Rigid walls

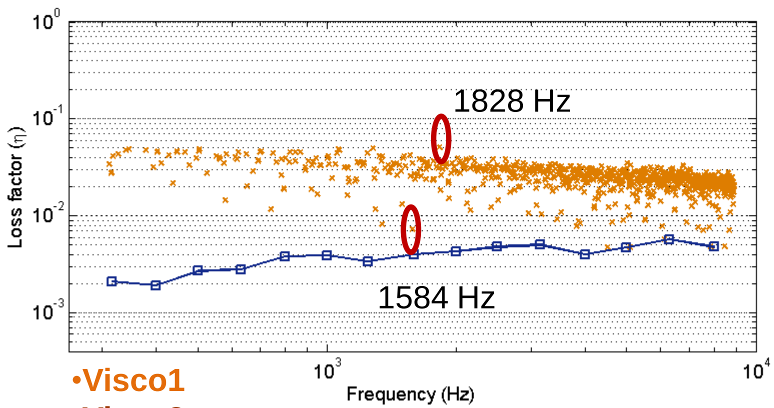
3rd STEP

Estimation of the subsystem modal damping

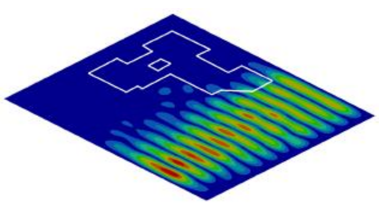
NASTRAN modal extraction with Lanczos Method

→ MSE & MSKE methods implemented in NASTRAN (DMAP)

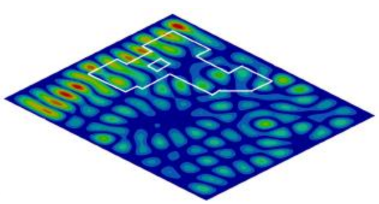
$(\omega_p^1, \Phi_p^1, \eta_p^1)$



- Visco1
- Visco2
- 1 mm steel



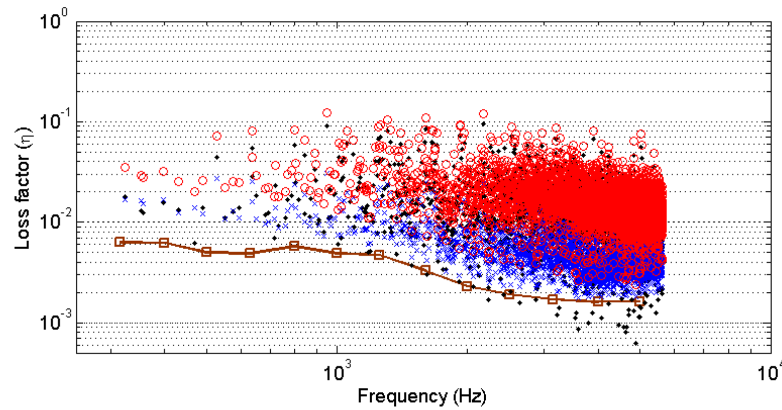
1584 Hz



1828 Hz

- Large variation between resonant modes
- Spatial deformation around the damping pad
- *Visc2* avg 40 % higher than *Visc1*

$(\omega_q^2, P_q^2, \eta_q^2)$

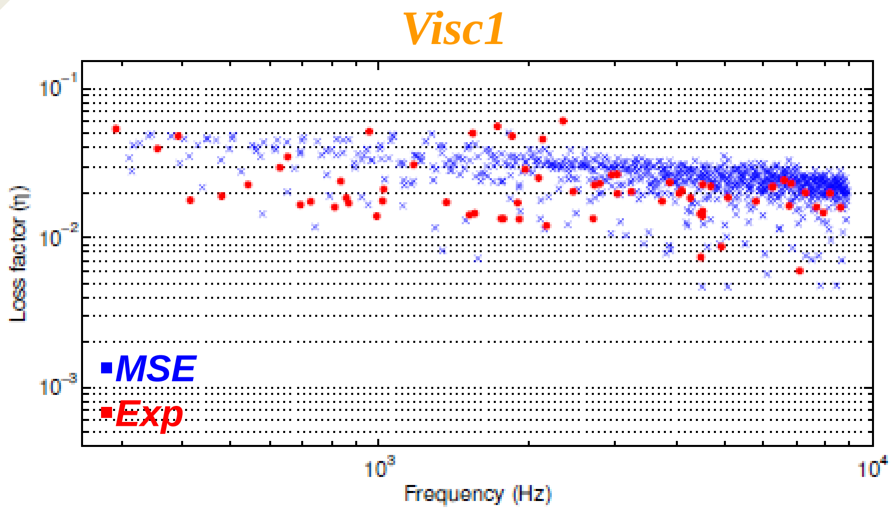


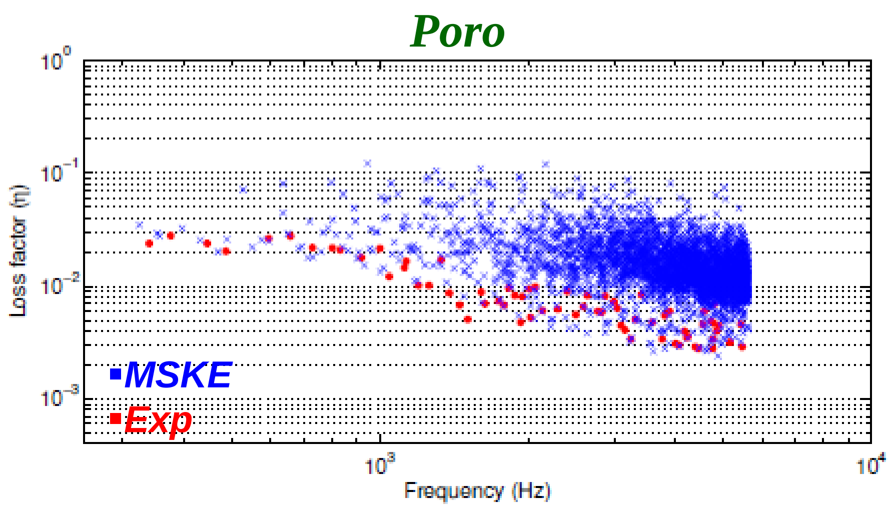
- η_n (total damping)
- $\eta_0\,(\Phi^T M_o\, \Phi^T) + \eta_{eq}(\Phi^T M_{eq} \Phi^T)$
- $[\chi_e/(1+\chi_e^2)] * [(\Phi^T K_{eq} \Phi^T)/\omega_n^2]$
- Empty Cavity

3rd STEP

Estimation of the subsystem modal damping

Experimental validation: High-resolution modal analysis based on ESPRIT algorithm





!!! For heavily damped modes, SNR is problematic experimentally.
➔ESPRIT effective for lightly damped modes!!!

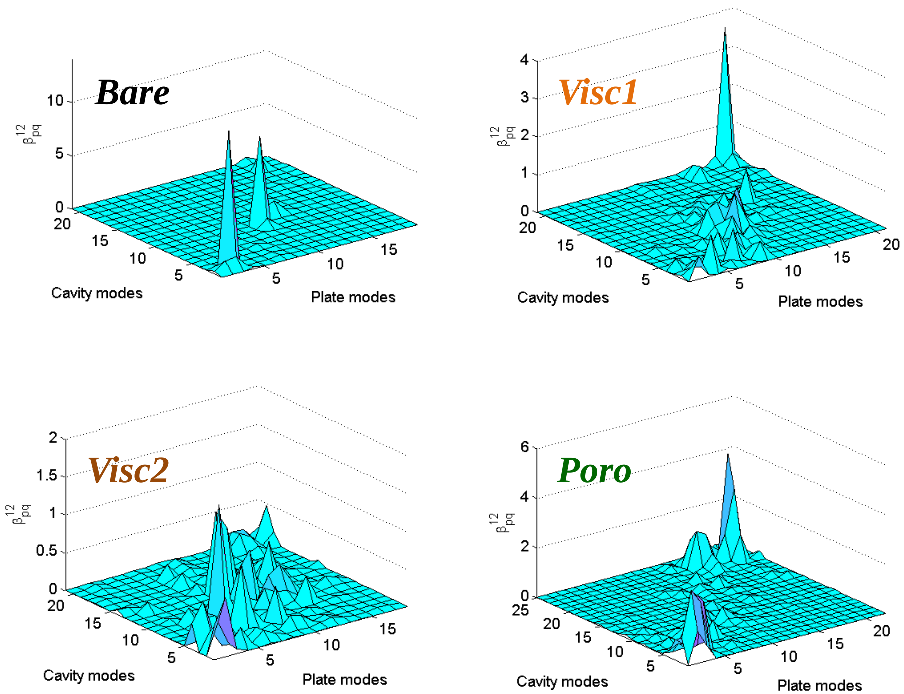
K. Ege, et al., High resolution modal analysis, *Journal of Sound and Vibration*, 325 (2009) 852-869

4th STEP

Computation of the Modal coupling factors

$$\beta_{pq}^{12} = \frac{W_{pq}^{12}}{M_p^1 \left(\omega_q^2\right)^2 M_q^2} \left\{ \frac{\omega_p^1 \eta_p^1 \left(\omega_q^2\right)^2 + \omega_q^2 \eta_q^2 \left(\omega_p^1\right)^2}{\left[\left(\omega_p^1\right)^2 - \left(\omega_q^2\right)^2\right]^2 + \left(\omega_p^1 \eta_p^1 + \omega_q^2 \eta_q^2\right)\left[\omega_p^1 \eta_p^1 \left(\omega_q^2\right)^2 + \omega_q^2 \eta_q^2 \left(\omega_p^1\right)^2\right]} \right\}$$

Modal coupling (β_{pq}^{12}) at 1 kHz



<u>5th STEP</u>

Computation of the subsystem energies

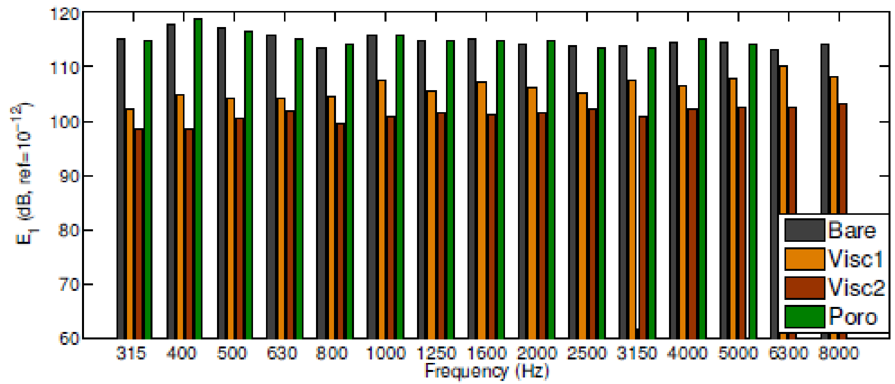
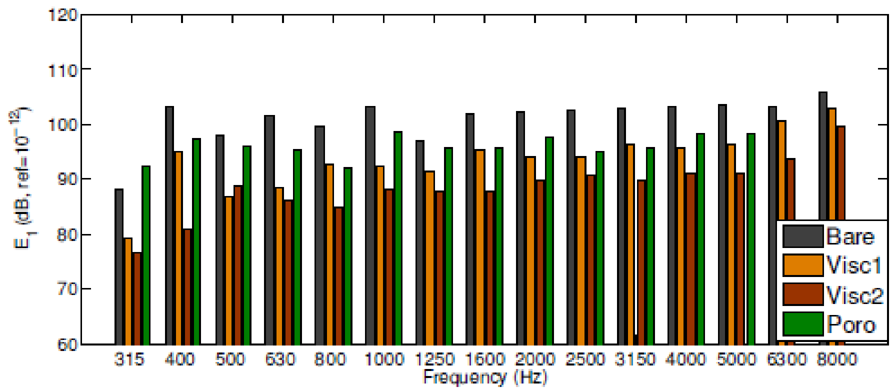
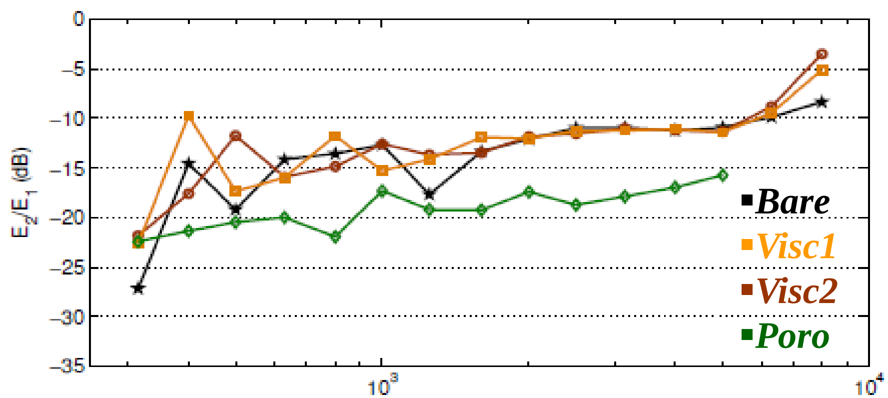


Plate Energies (1/3 octave)

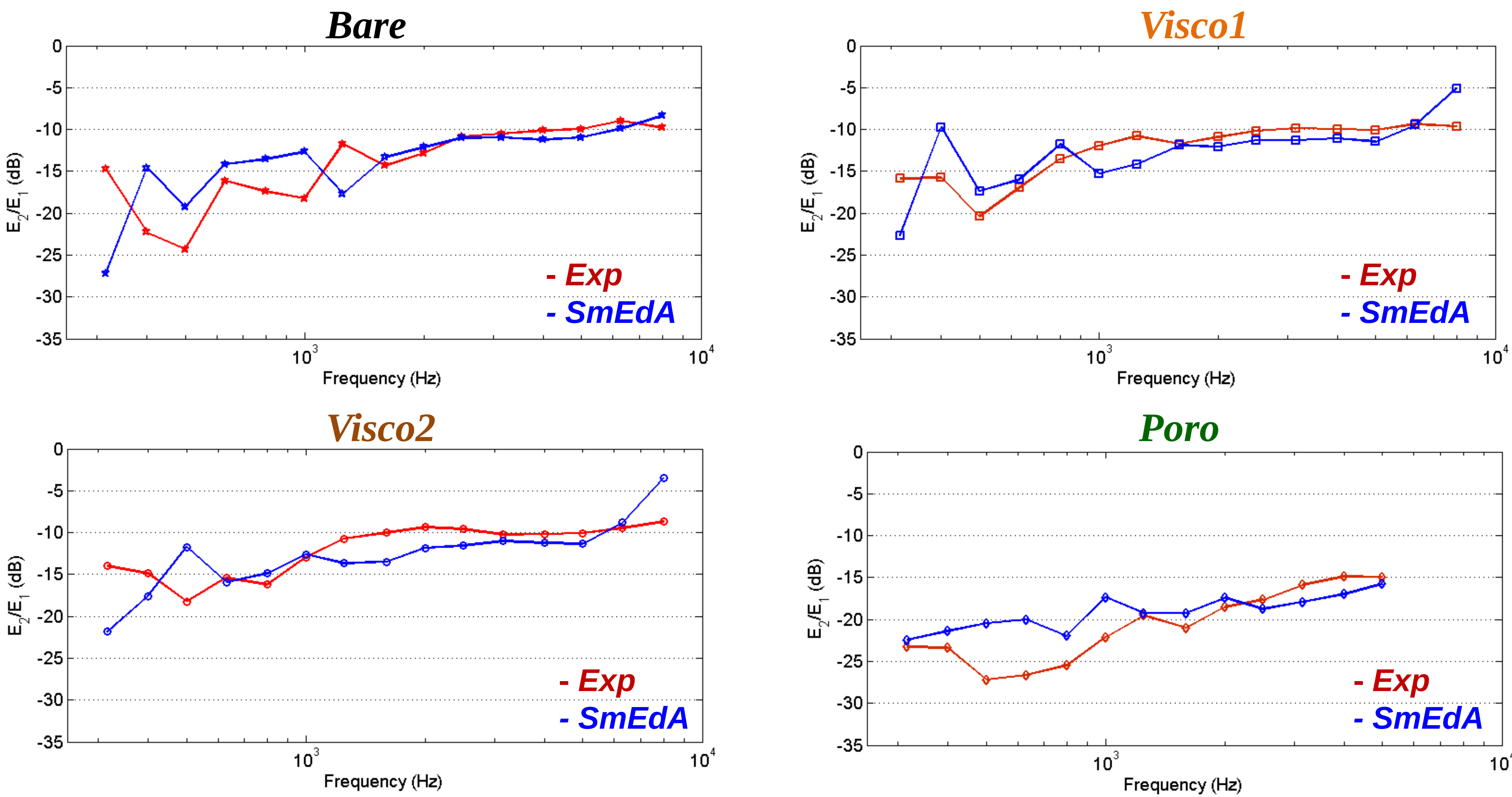


Cavity Energies (1/3 octave)



Energy Ratio (E_2/E_1) (1/3 octave)

Experimental Validation: Subsystem energy ratio (E_2/E_1)

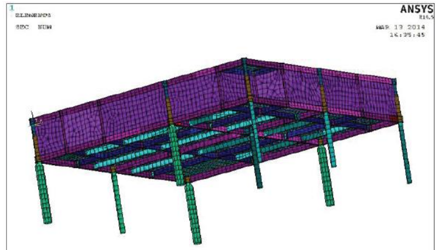


At low frequency, discrepancies due to the boundary condition?

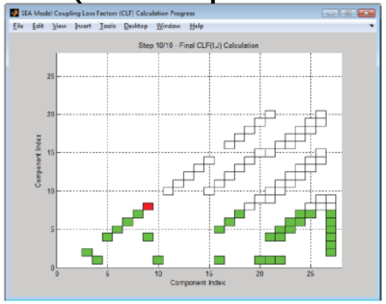
Next future: Application to the truck cab with trims (Y. Gerges)

Recent works achieved by ACOEM (acoustic consulting company):

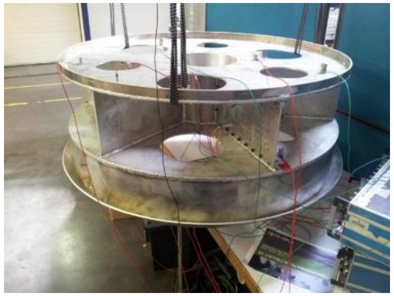
- Automation of the numerical process through an in-house code (developed under ANSYS-APDL environnent)

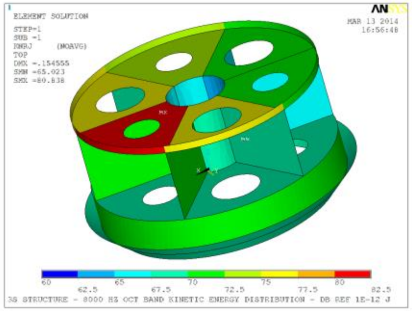


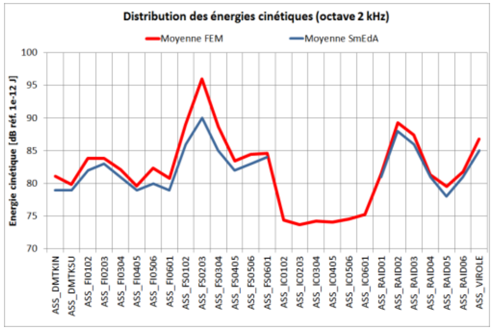
Oil rig model



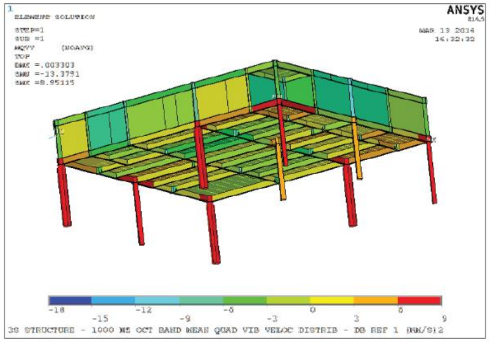
- Benchmark on a mock-up of a nuclear power plant structure







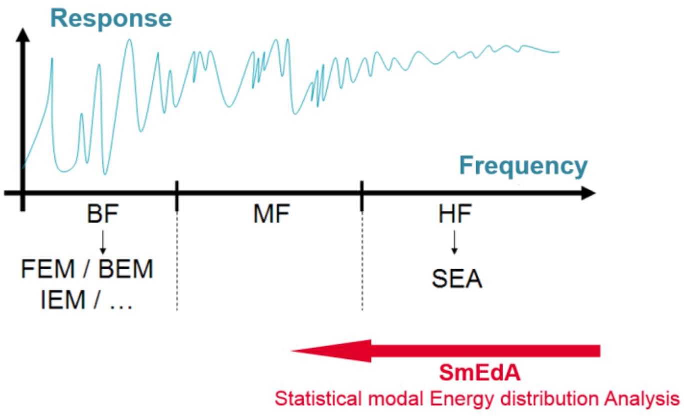
- Applications on industrial buildings



P. VOUAGNER, L. MAXIT, C. THIRARD, C. DESLOT, J.L. GUYADER, Modélisation de la propogation du bruit solidien dans les structures industrielles. CFA 2014, Poitier, France, April 2014.

SmEdA overview

- SEA energy equipartition assumption relaxed
    - ➔Extension to low modal overlap subsystems
    - ➔Extension to non-homogeneous subsystems (with local energy description)
- Method based on the uncoupled subsystems modes;
    - ➔Description of subsystems with complex geometry/mechanical properties (using FEM)
    - ➔Description of dissipative treatments
- Non-resonant transmission modelling
    - ➔ Prediction of TL of complex structures in mid-frequency
- Hybrid SEA/SmEdA model



Future research on SmEdA

- Uncertainty (screening technique, propagation of uncertainties, variance estimation,…)
- Tools for energy transfer analysis (using the graph theory: tomorrow, Oriol Guasch,…)

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Thank you for your attention